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Preliminary operating models for the 3LN redfish management strategy evaluation

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Introduction

The Commission has requested a review and update of the Management Strategy Evaluation (MSE) for the NAFO division 3LN redfish stock ([SCS, 2021](#)). To date, a thorough data review has been conducted ([Perreault et al., 2022](#)) and the data that will inform the MSE process have been finalized ([Perreault and Rogers, 2023](#)). This document presents a progress update on the preliminary development of operating models for 3LN redfish. Note that models described below are not yet finalized.

Methods

Surplus Production Models

Surplus production models (SPMs) are a common approach when reliable age and size information are not available, and are the simplest stock assessment models that can produce a full suite of management advice ([Haddon, 2010](#)). Conceptually, SPMs pool all the effects of growth, mortality, and recruitment into a production function, and relate them to aggregate biomass and harvest rates. SPMs are composed of two parts: the process equations describing how the biomass changes over time, and the observation equations that relates the survey index to the biomass. Historically, due to computational restrictions only observation errors or process errors were estimated (e.g. [Prager, 1992](#); [Schnute, 1977](#)). More recently, state-space SPMs that can account for both observation and process error have become computationally feasible and have been implemented in a wide variety of management settings (e.g. [Pedersen and Berg, 2017](#); [Winker et al., 2018](#)).

Two types of SPMs are considered for the 3LN redfish MSE: a state-space surplus production model in continuous time (SPiCT, [Pedersen and Berg, 2017](#)) and a bespoke state-space surplus



production model (SSSPM). Surplus production models were developed in order to compare to the approach used for the previous MSE process (e.g., [Dauphin et al., 2014](#)). Additionally, the state-space setting of the selected SPMs allow for errors in the underlying population processes, which may better capture time-varying processes (e.g. sporadic recruitment) than the previous model formulations.

Data inputs

A suite of SPMs are in progress for 3LN redfish, however for simplicity, the results presented here are for one model formulation. Table 1 provides a summary of the models explored up to now (e.g. divisionally disaggregated). For the models discussed in the document, the data used are (see [Perreault and Rogers, 2023](#) for details):

- Landings (Kt; 1960-2022)
- Converted stratified RV biomass estimates (Historic) for new index strata
 - 3L Canadian Fall Engel (1975-1982), Converted (1990-2020)
 - 3L Canadian Spring Yankee (1975-1982), Engel (1985-1991), Converted(1992-2019)
 - 3L EU-Spain Converted (2004-2019)
 - 3N Canadian Fall Converted (1993-2020)
 - 3N Canadian Spring Yankee (1977-1982), Engel (1984-1990), Converted (1991-2019)
 - 3N EU-Spain Converted (1996-2021).

Note that the landings time series used in the models begin in 1960 and not 1959 since early model runs indicated that the model was sensitive to the first year of data (see Supplementary Materials Figs 23 & 24). Additionally, the data prior to 1960 were extracted from pdfs of the historic Statistical Bulletins and not from the STATLANT 21A database (see [Perreault et al., 2022](#) for details), and we speculate that the uncertainty around the 1959 estimate may differ from the estimates from 1960 onward. Therefore, we use 1960 as the first year in our landings series.

SPiCT

The SPiCT model provides a flexible framework that treats biomass and fishing mortality rates as unobserved processes (i.e. process equations), and allows for observation errors in both the surveys and commercial catches (i.e. observation equations). The SPiCT model has been used in fisheries management worldwide (e.g. [González Herraiz et al., 2023](#); [Rigét et al., 2022](#)) and can be easily implemented using the related Rpackage (`spict`, [Pedersen and Berg, 2017](#)).

Parameters are estimated either in a Bayesian or frequentist setting, by minimizing the negative log-likelihood function in the former, or using informative priors which are multiplied by the likelihood function to extract the posterior distribution in the latter. For brevity, we do not discuss the model here in detail, but an overview is given in Appendix A. The SPiCT package can provide estimates of both deterministic and stochastic reference points (see [Pedersen and Berg, 2017](#) for details). Our results are based on the stochastic reference points since these are recommended in practice.

We fit preliminary SPiCT models for landing data from 1960-2021 and truncated landing data from 1975-2021, the minimum year of our survey data, in order to assess the influence of extending the model to years where no survey data are available. The models presented here are for divisionally aggregated biomass estimates (3LN), although we also fit models separately by division (Table 1). For our preliminary model runs, we fixed $n=2$, which reduces the Pella-Tomlinson model to the classic Schaefer model. No priors were used in our model formulations.

Model fit was assessed using the checklist provided by ICES ([ICES, 2017](#)), and include ensuring that the model has converged, checking one-step-ahead (OSA) residuals and related assumptions (bias, autocorrelation and normality), examining retrospective plots, checking the shape of the production curve, assessing whether the assessment uncertainty was too high, and a jitter self-start test. Note that for ease of readability, all model diagnostic outputs are provided in the Supplementary Materials.

SSSPM

We also develop a bespoke state-space surplus production model (SSSPM) in order to have a flexible framework, should non-standard model formulations need to be investigated. As in the SPiCT model, we allow for errors in the underlying population processes, and in the observations (see Appendix B for model details). Parameters were estimated using the frequentist approach based on the marginal negative log likelihood, which was derived via the Laplace approximation in TMB ([Kristensen et al., 2015](#)). Fixed effect parameters were estimated using the `optim` function in R ([R Core Team, 2022](#)).

We fit preliminary SSPMs for landing data from 1960-2021 and truncated landing data from 1975-2021, the minimum year of our survey data. The models presented here are for divisionally aggregated biomass estimates (3LN), although we also fit models separately by division (Table 1) for unconverted and converted survey data. Due to issues getting the model to converge, the observation model standard deviation was fixed at 0.2. Sensitivity runs to this assumption will be conducted for the base OMs selected. All other parameters were freely estimated.

Model fit was assessed similarly to the SPiCT model, by ensuring that the model has converged, checking residuals and related assumptions (bias, autocorrelation and normality), checking the shape of the production curve, and assessing whether the assessment uncertainty was too high. Due to time constraints the jitter self-start and retrospective runs are in progress.

Age structured catch at length model

Surplus production models are known to perform poorly when there are time varying changes in a variety of population processes, including recruitment and growth ([Punt and Szuwalski, 2012](#)). Recruitment success is poorly understood for redfish therefore surplus production models may not adequately capture stock dynamics. As such, we consider an integrated state-space age-structured catch-at-length (ACL) model for 3LN redfish that utilizes the available information on fishery total catch-at-length and abundance indices-at-length, which have not been considered in previous assessment models. The ACL model is based on [Cadigan et al., 2024](#), and can provide estimates of recruitment deviations and separate catchability parameters at length for each survey, and was therefore expected to better model stock dynamics and its related uncertainties.

Data inputs

As for the surplus production models, the results presented are for a subset of model formulations using the data detailed below (see [Perreault and Rogers, 2023](#) for details).

- Converted RV abundance at length for new index strata (Historic)
 - 3LN Canadian Fall Engel (1984-1989), Converted (1991-2020)
 - 3LN Canadian Spring Engel (1985-1990), Converted (1997-2019)
 - 3L EU-Spain Converted (2004-2019)
 - 3N EU-Spain Converted (1997-2019)

- Commercial catch at length
 - All (1975-2021)
- Weight at length
 - Portuguese commercial sampling (3LN; 1990-2019), EU-Spain RV 3L (1997-2021) and 3N (2003-2019)

The models presented here are for divisionally aggregated biomass estimates (3LN), although we also fit models separately by division (Table 1) for unconverted and converted survey data. Due to confounding between the L_∞ and k parameters in the von Bertalanffy equation, L_∞ was fixed at 42cm, which seemed reasonable for preliminary model explorations given the sply plots (Supplementary Materials Fig. 25) and previous analyses (Cadigan and Campana, 2017). The logit parameters for fishing mortality rates, i.e. ϕ_{FA} , ϕ_{FY} were fixed at 0.8 and 1.7, respectively, based on early model runs. This also ensured that the fishing mortality rates at age and year varied smoothly over time.

Model fit was assessed by ensuring that the model has converged, checking residuals and related assumptions (bias, autocorrelation and normality) and assessing whether the assessment uncertainty was too high. Due to time constraints the jitter self-start and retrospective runs are in progress.

Results and Discussion

Surplus Production Models

Overall, the surplus production models, no matter the baseline formulation (SPiCT or SSSPM) or starting year (1960 or 1975) fit the data well (Supplementary Materials Figs. 7, 14, 18 & 22) and provided similar estimates of population processes (Table 2). Both the SPiCT and SSSPM models were generally insensitive to starting the time series in 1960 or 1975 (Table 2), with the SPiCT model less sensitive than the SSSPM. Estimates of r and K were similar for both model formulations, although the K predictions were larger and r smaller for the SPiCT model than for the SSSPM.

ACL Model

The simple ACL mode fit the data well with no large patterns present in the residuals across lengths or years for the surveys or the commercial catch (Supplementary Materials Figs. 26 & 27). However, no matter the underlying model formulation explored (Table 3), the ACL model required large process error standard deviation, σ_{pe} to fit the survey indices and landings (Table 3; much larger than what is typically seen in state-space stock assessment models). Smaller values of σ_{pe} resulted in a much worse fit to the data, which indicates that the reported catches and our assumptions about M have not described redfish population dynamics well, as indicated by the survey indices. This, combined with the the large σ_I estimates (Fig. 9) indicates that even with a large amount of process error, there is substantial lack of fit to the survey indices.

Additionally, catchability estimates were strange (Fig. 10) and suggested that the surveys are not seeing larger fish, which is unexpected for a bottom-trawl survey. Sensitivity runs with q forced to be flat topped (Fig. 10) do not appear to address this issue, which may further indicate a lack of fit to the survey indices.

Summary and Future Directions

Preliminary surplus production models fit the data well, although we do have some concerns about basing an MSE on a model that is known to perform poorly when there are potential time varying processes in population dynamics. As such, we will continue to investigate fitting models to the length composition data and a simpler SURBA model is currently in progress (e.g., [Kumar et al., 2020](#)). Preliminary investigations have been promising, including early considerations of including environmental variables in the process equations to potentially better capture shifts in the underlying population that the current models cannot (e.g., [Stock and Miller, 2021](#)).

Figures

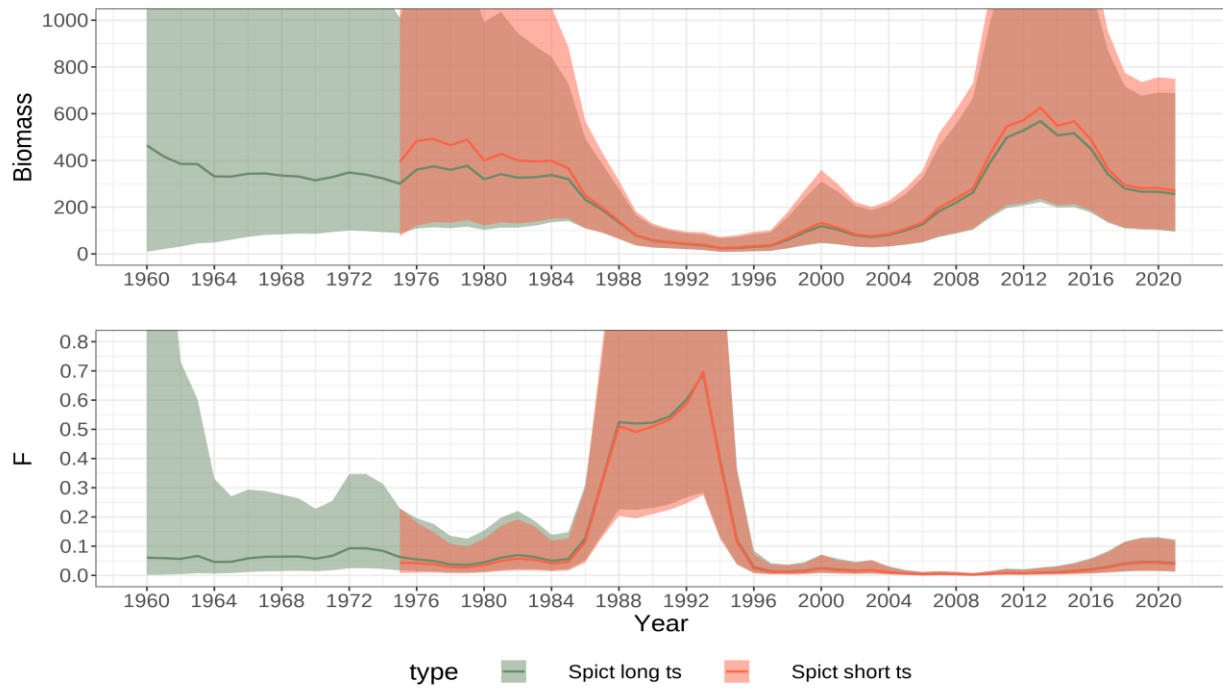


Figure 1. SPiCT estimates from the full (green) and truncated (orange) time series for biomass (top) and average fishing mortality rates (bottom).

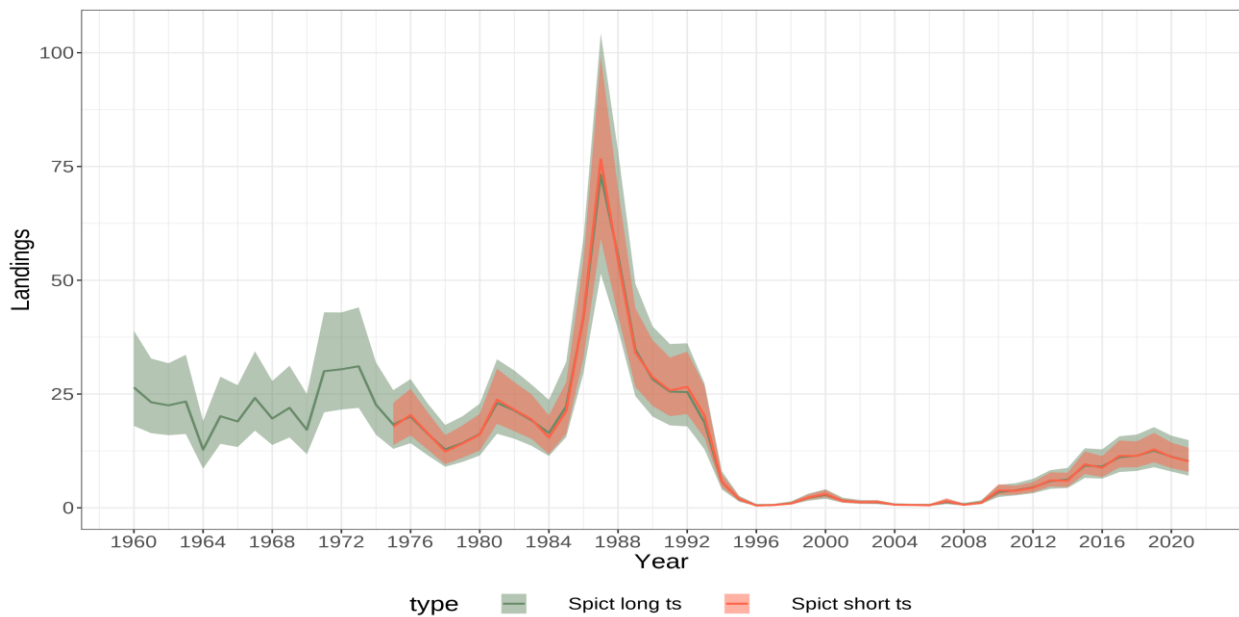


Figure 2. SPiCT landings estimates from the full (green) and truncated (orange) time series.

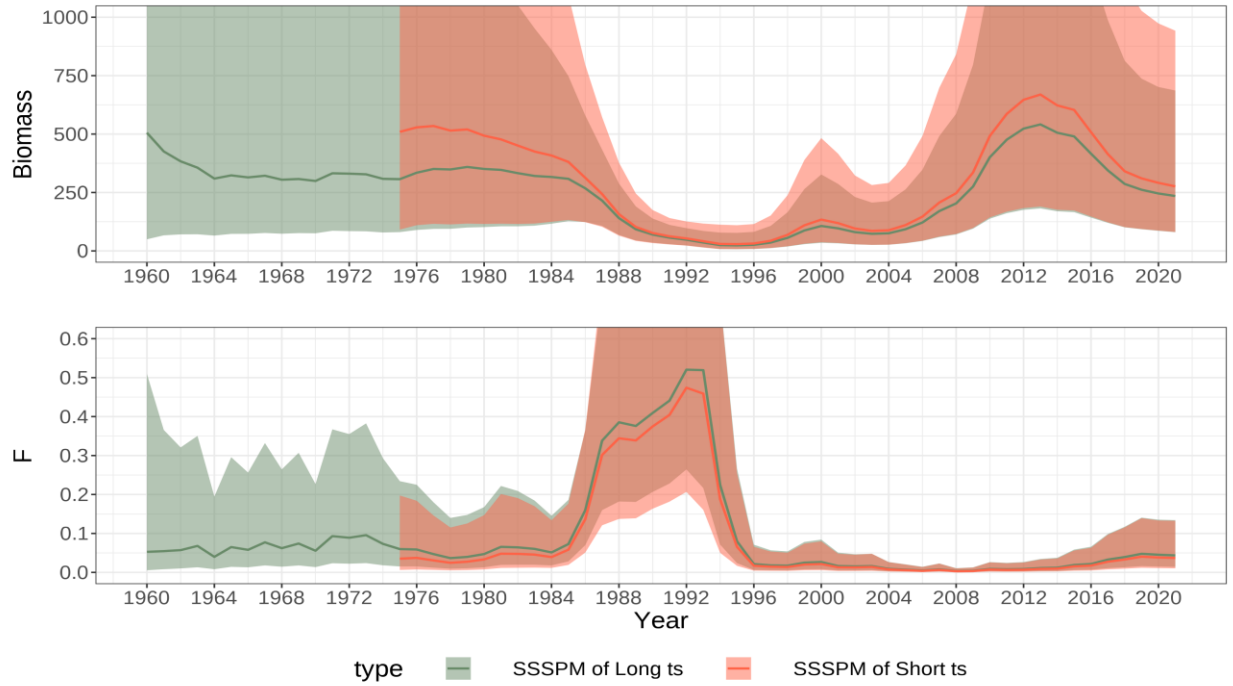


Figure 3. SSSPM estimates from the full (green) and truncated (orange) time series for biomass (top) and average fishing mortality rates (bottom).

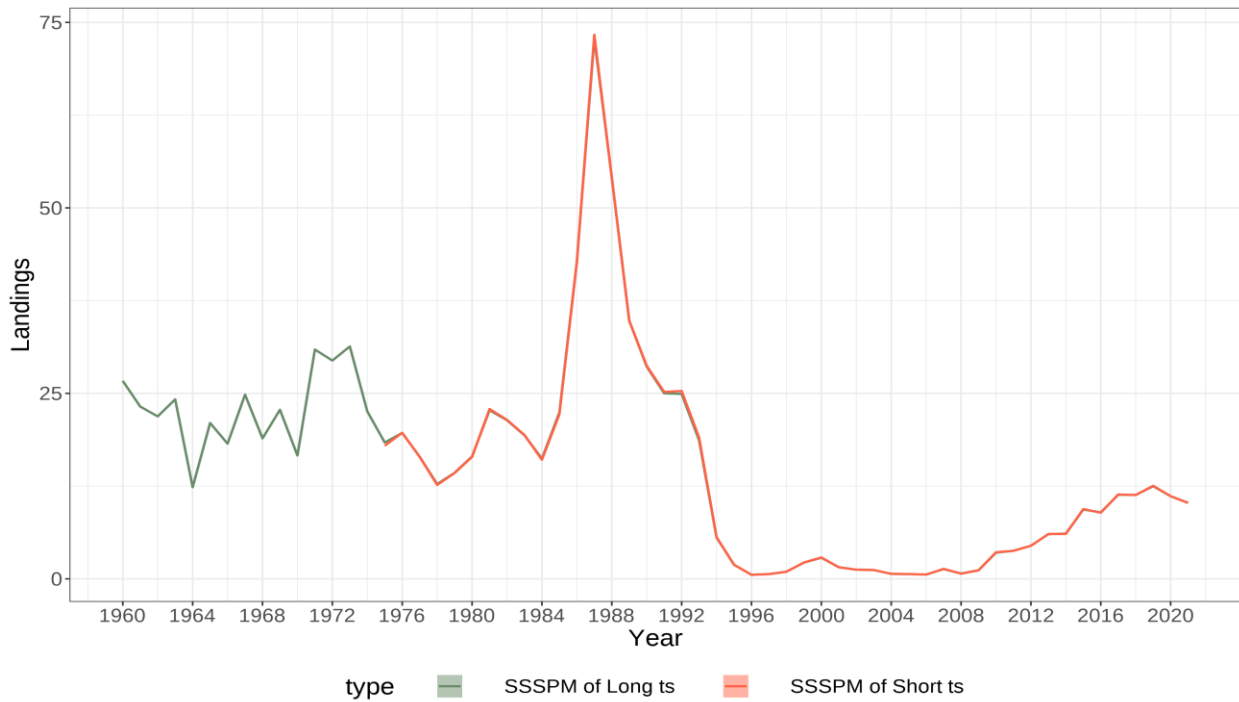


Figure 4. SSSPM landings estimates from the full (green) and truncated (orange) time series.

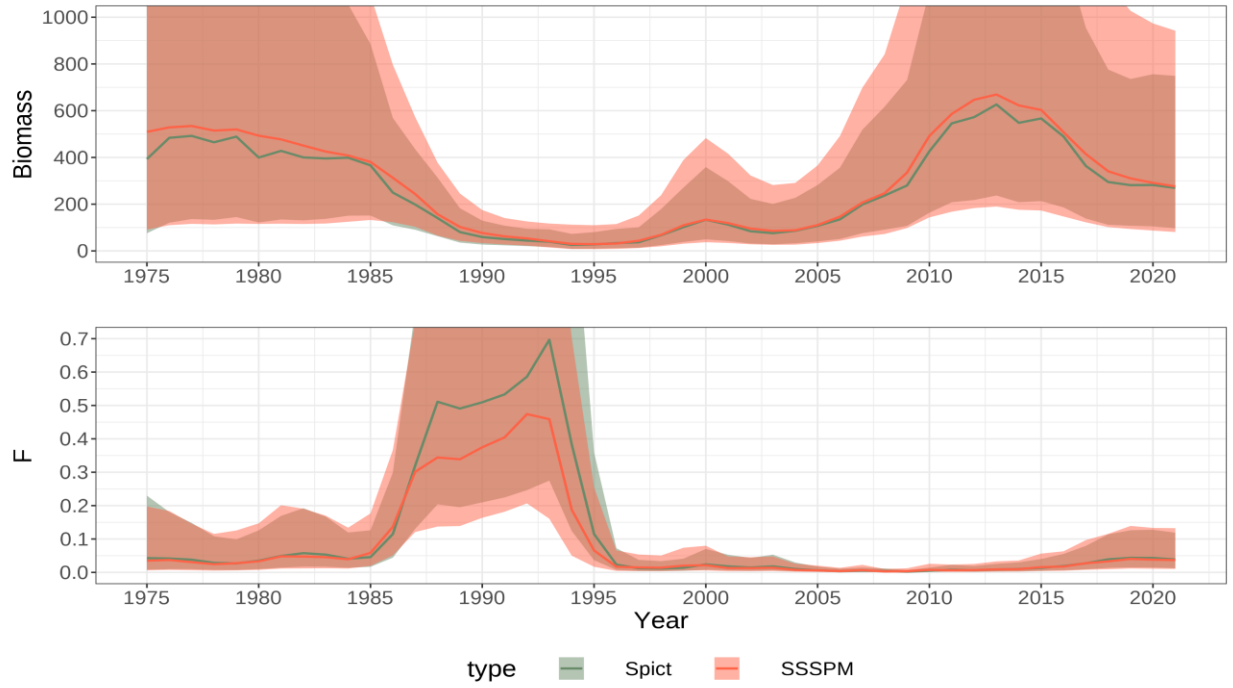


Figure 5. Estimates from the SPiCT (green) and SSSPM (orange) time series for biomass (top) and average fishing mortality rates (bottom).

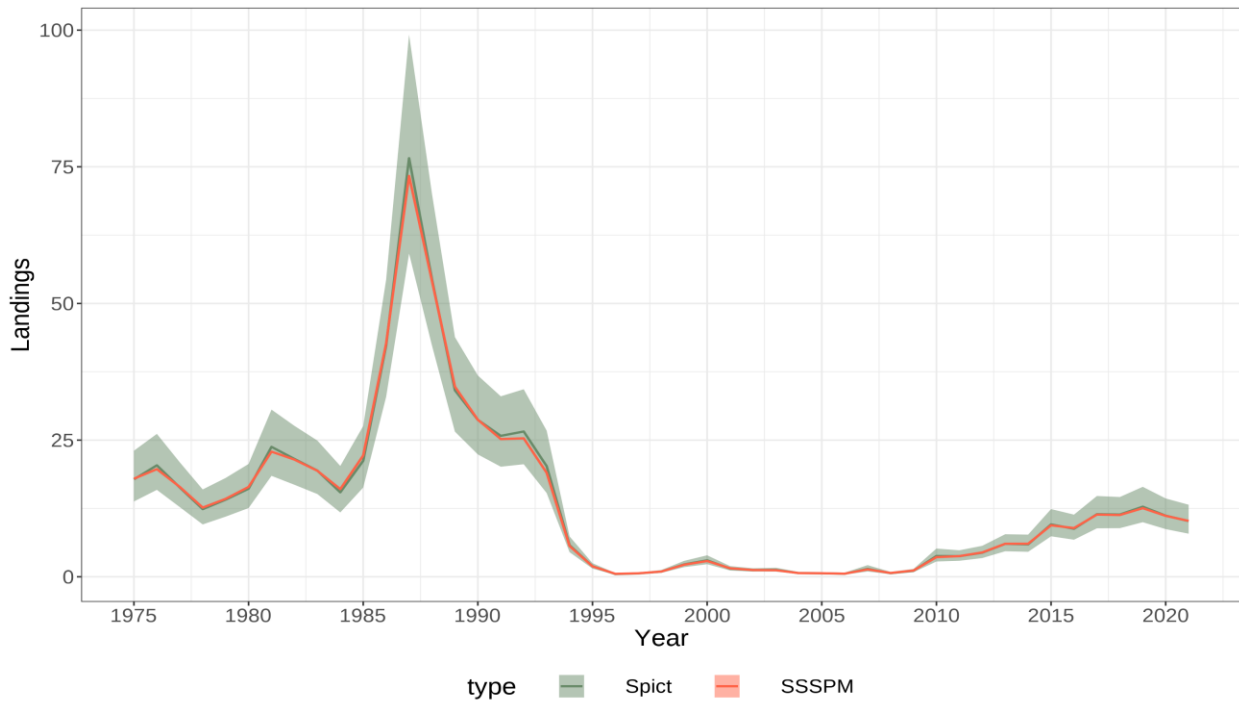


Figure 6. Landings from the SPiCT (green) and SSSPM (orange) time series.

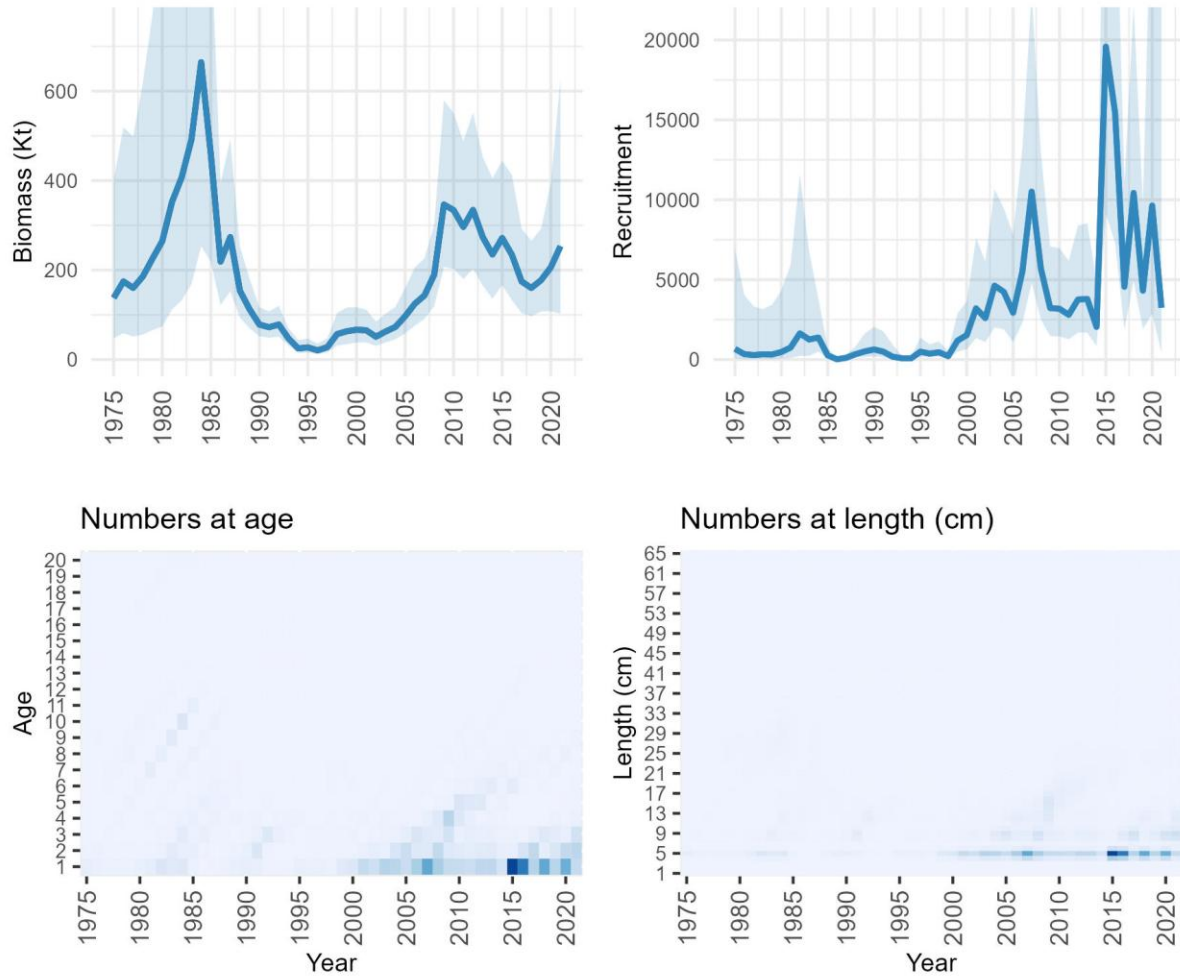


Figure 7. ACL model predictions of biomass (top left), recruitment (top right), natural mortality rate (bottom left) and harvest rates (bottom right). The blue shaded regions are the 95% confidence intervals.

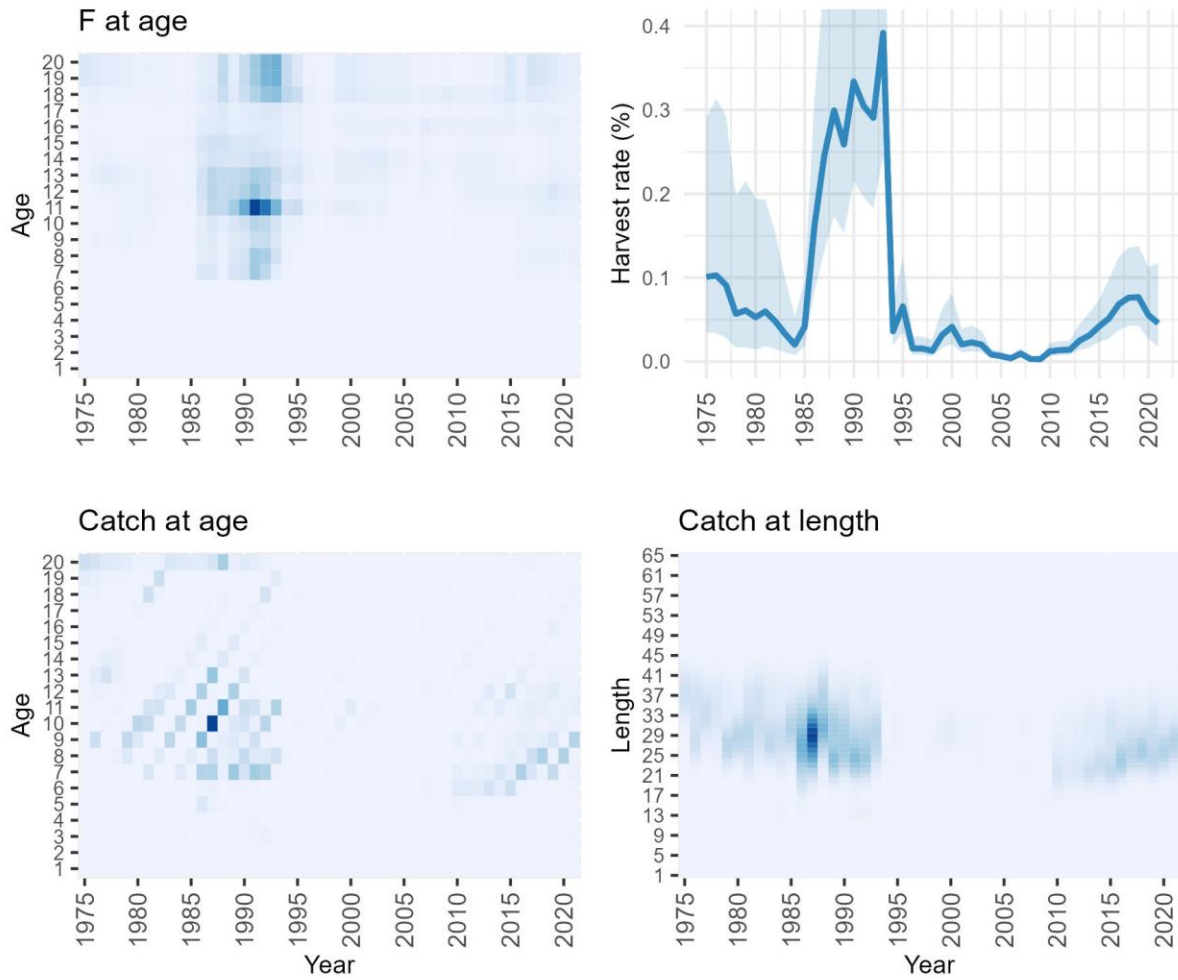


Figure 8. ACL model predictions of fishing mortality rates at age (top left), harvest rates (top right), catch at age (bottom left) and catch at length (bottom right). The blue shaded regions are the 95% confidence intervals.

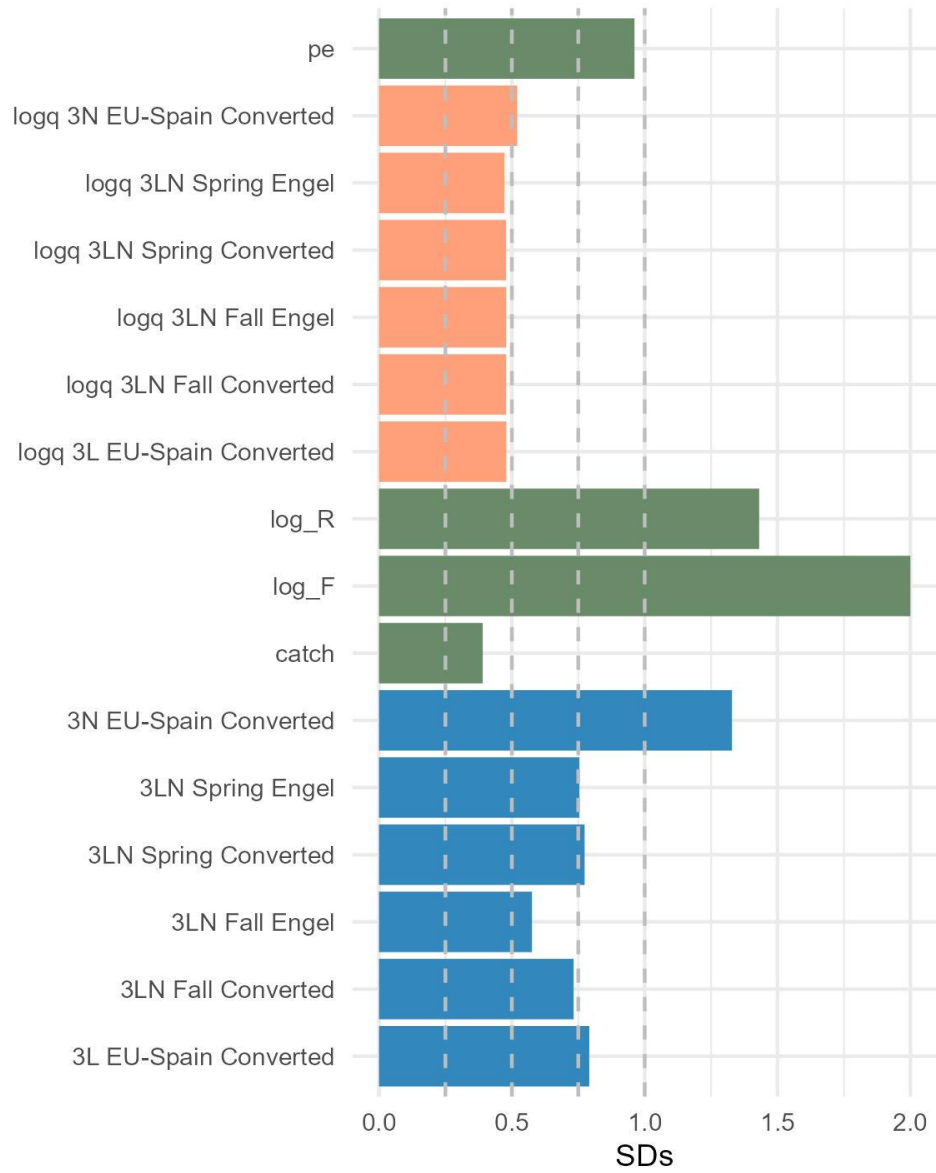


Figure 9. Standard deviation estimates from the ACL model.

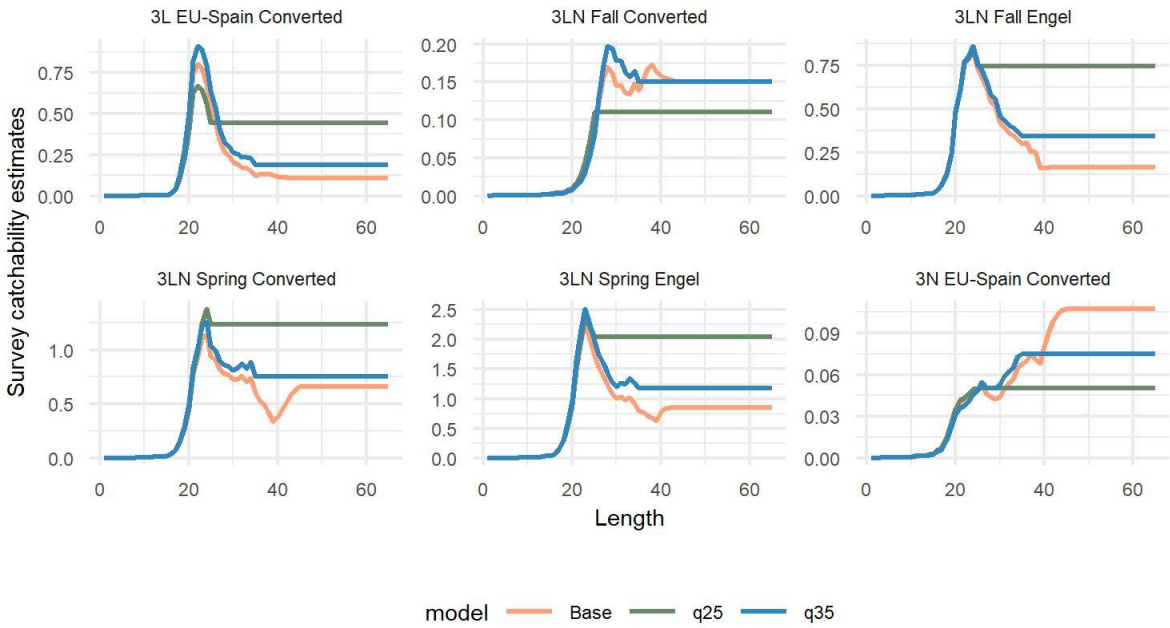


Figure 10. Catchability estimates from the ACL for model runs with the max catchability estimate fixed at 25 (q25; green), 35 (q35; blue) and 45 (base; orange) centimeters.

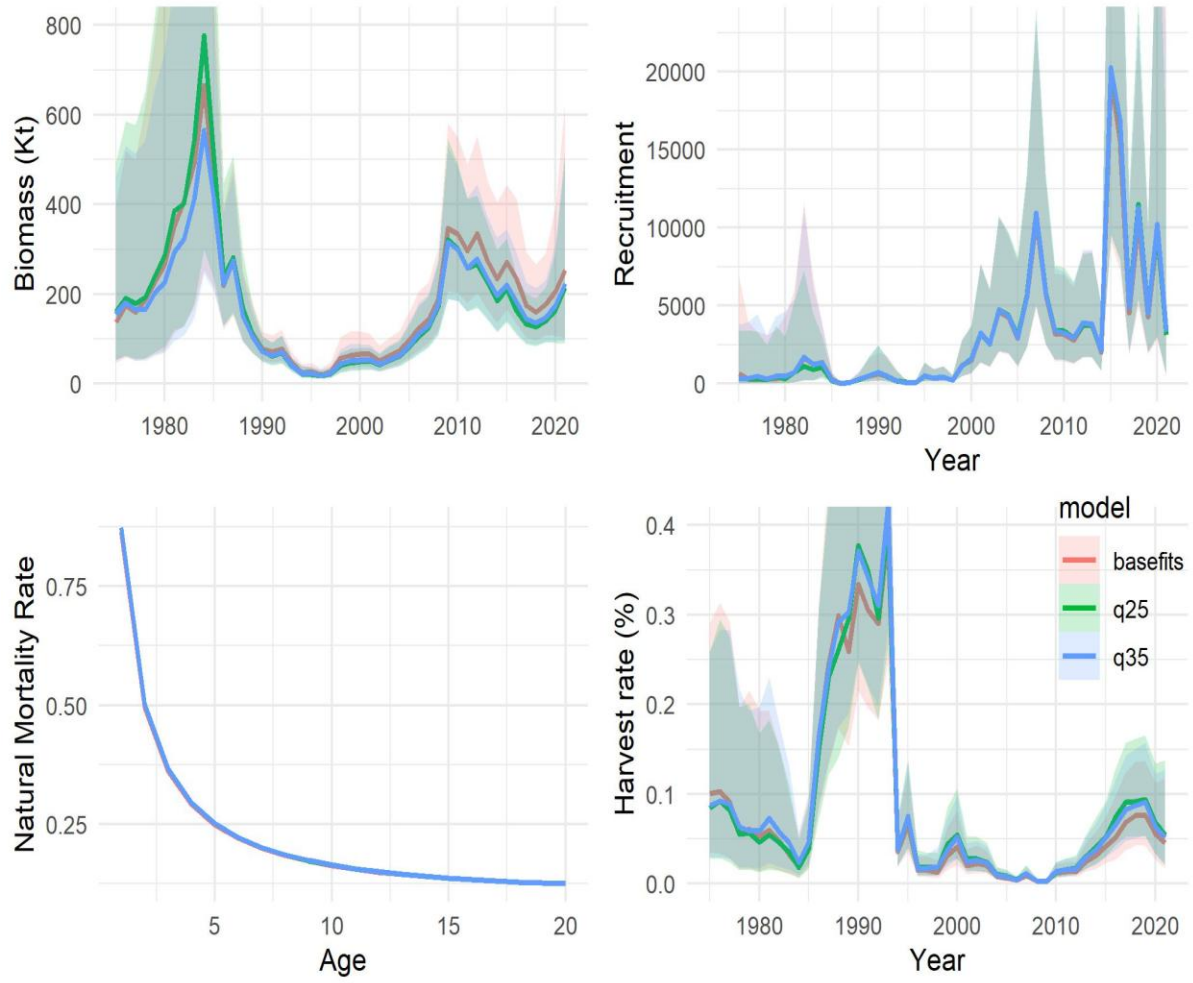


Figure 11. Comparison of ACL predicted populations processes from base model (pink), fixing the max catchability estimate at 25 (q25) and 35 (q35) centimeters.

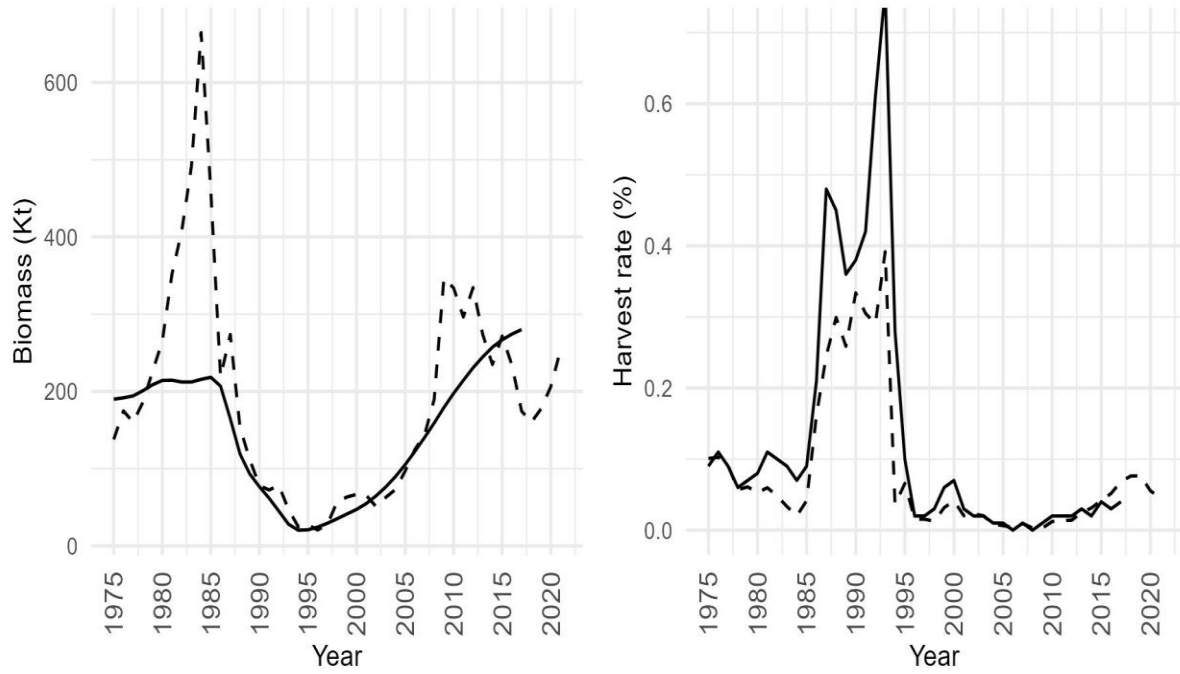


Figure 12. Comparison of ACL model predictions from the ASPIC (solid) and ACL (dashed) models.

Tables

Table 1. General summary of model formulations explored and/or for future consideration

Assumption	Variations
Landings start	1960, 1975
Data type	Converted, Unconverted
Divisions	Aggregated (3LN), Disaggregated (3L,3N) Wider (3LNO)
Depth	Deep/shallow

Table 2. Summary of surplus production model runs and parameter estimates

Model	-	SPiCT1	SPiCT2	SPiCT3	SSSPM1	SSSPM2
Start Year(Landing)		1959	1960	1975	1960	1975
Convergence		No	Yes	Yes	Yes	Yes
Fixed parameters		n	n	n	n	n
Dignostic Problems	OSA Residuals		No	No		
	ACF		No	No		
	SAMPLE Quantiles		Yes	Yes		
Relative reference points B2021/Bmsy		1.362	1.369	1.369	1.377	1.438
Biomass2021(000s)		54.188	255.911	269.404	234.892	276.079
Relative reference points F2021/Fmsy		0.369	0.249	0.245	0.372	0.356
Robustness to Initial			No	Yes		
r		1.22	0.366	0.372	0.235	0.208
K(000s)		117.897	448.32	484.578	341.124	384.084
Sdb**		0.619	0.308	0.339	0.732***	0.725***
AIC/nll		621.79	590.76	568.97	261.42	250.2
q Converted _ 3N _ summer		1677.605	343.393	318.423	237.924	197.294
q Converted _ 3L _ summer		1104.003	222.376	206.043	196.734	162.569
q Converted _ 3L _ fall		1044.542	232.823	216.525	229.398	188.8
q Converted _ 3N _ fall		2574.969	543.01	504.163	570.963	471.289
q Engel _ 3L _ fall		1289.164	206.444	193.919	261.776	217
q Converted _ 3L _ spring		946.349	195.71	181.719	355.089	292.608
q Converted _ 3N _ spring		1220.068	259.369	240.819	172.35	149.232
q Engel _ 3L _ spring		304.569	56.944	54.228	52.322	46.508
q Engel _ 3N _ spring		101.349	19.413	18.146	18.41	16.062
q Yankee _ 3L _ spring		775.088	54.819	40.533	52.633	36.281
q Yankee _ 3N _ spring		251.774	15.699	12.177	15.334	10.934
Catch_2022 (000s)		12.256	10.6719413	10.6388		
E(B_inf)****		54.363	259.354	249.389		

**standard deviation of biomass process error

***standard deviation of process error

****equilibrium biomass under current conditions

Table 3. Summary of ACL model runs and parameter estimates

Parameter	Basefit	q35	q25
meanR	1,124.079	1,132.940	1,212.460
meanR.1	1,058.049	1,058.304	982.508
std_R	1.431	1.392	1.374
std_3L_EUS_C	0.791	0.776	0.854
std_3LN_Fall_C	0.733	0.707	0.733
std_3LN_Fall_E	0.576	0.629	0.661
std_3LN_Spr_C	0.774	0.776	0.804
std_3LN_Spr_E	0.754	0.805	0.895
std_3N_EUS_C	1.328	1.307	1.351
logit_phi_resid	0.807	0.799	0.822
std_catch	0.390	0.399	0.427
logit_phi_catch	0.587	0.577	0.663
qstd_3L_EUS_C	0.479	0.482	0.456
qstd_3LN_Fall_C	0.479	0.480	0.452
qstd_3LN_Fall_E	0.479	0.478	0.429
qstd_3LN_Spr_C	0.478	0.477	0.444
qstd_3LN_Spr_E	0.473	0.471	0.414
qstd_3N_EUS_C	0.520	0.525	0.539
std_pe	0.961	0.868	0.869
logit_R	0.522	0.526	0.553
vbk	0.109	0.107	0.107
cv_len	0.092	0.091	0.091
std_F	2.000	2.000	2.000
F_main	0.094	0.093	0.084
F_main.1	0.095	0.096	0.089

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Appendix A: SPiCT model

Process model

The SPiCT model is based on the traditional Pella-Thomlinson biomass dynamics model that includes process error,

$$dB_y = \left(\gamma m \frac{B_y}{K} - \gamma m \left[\frac{B_y}{K} \right] - F_y B_y \right) dy + \sigma_B B_y dW_y$$

where B_y is the exploitable biomass at time y , K is the carrying capacity, m is the productivity parameter, and $\gamma = \frac{n^{n/(n-1)}}{(n-1)}$. For our preliminary applications, and for comparative purposes to the SSSPM, we fix $n = 2$, which reduces the Pella-Tomlinson model to the classic Schaefer model ([Schaefer, 1954](#)). Unlike most SPM approaches, the SPiCT model treats the fishing mortality rates F_y as an unobserved process, which allows estimation of F_y at any time, even if catch observations are not available, i.e.,

$$F_y = S_y G_y,$$

where S_y and G_y are the seasonal and random components, respectively. For our application, we use annual catch data, so $S_y = 1$. The random process is defined in terms of standard Brownian motion (see [Pedersen and Berg, 2017](#) for details),

$$d\log G_y = \sigma_F dV_y,$$

where σ_F is the standard deviation of the random noise term and dV_t is the defined Brownian motion.

Observation model

The observation equations are defined in terms of the commercial catch, which is reported as the cumulative catch C_y over a time interval Δ_y ,

$$\log(C_y) = \log \left(\int_y^{y+\Delta_y} F_s B_s ds \right) + \epsilon_{c,y}$$

where $\epsilon_y \sim N(0, \sigma_C^2)$. The survey indices are treated as a snapshot in time, i.e.,

$$\log(I_i) = \log(q_i B_y) + \epsilon_{s,y,i}$$

where $\epsilon_{s,y,i} \sim N(0, \sigma_I^2)$ are the index observation errors, with catchability parameters q_i estimated for each survey.

Appendix B: SSSPM

Process model

The bespoke SSSPM is based on the Schaefer surplus production model formulation,

$$B_{y+1} = B_y + rB_y \left(1 - \frac{B_y}{K}\right) - C_y,$$

for biomass B_y , carrying capacity K , and catch C_y , which can be rewritten as

$$P_{y+1} = P_y + rP_y(1 - P_y) - H_yP_y,$$

where $B_y = H_yP_yK$, $P_y = \frac{B_y}{K}$, and $C_y = H_yB_y$. In the state-space setting, the process equations are then

$$P_{y+1} = (P_y + rP_y(1 - P_y) - H_yP_y)\exp^{\epsilon_y},$$

$$H_{y+1} = H_y\exp^{\delta_y},$$

where ϵ_y are the process errors, $\epsilon_y \sim AR(1)$ with parameters $\phi_\epsilon, \sigma_\epsilon$ to estimate. The harvest rate deviations, δ_y , are assumed more similar closer together in time, i.e., $\delta_1, \dots, \delta_Y \sim N(0, \sigma_\delta^2)$ with σ_δ to estimate.

Observation model

The survey index observation equation relates the underlying population processes to the observations,

$$I_y = q_s B_y \exp^{\epsilon_{sy}},$$

with $\epsilon_{sy} \sim N(0, \sigma_s^2)$. Similarly, the catch observation equation is given by

$$C_y = H_y K P_y \exp^{\epsilon_{cy}},$$

with $\epsilon_{cy} \sim N(0, \sigma_c^2)$.

Appendix C: Age Structured Catch-at-Length Model

Process model

The ACL model is based on the standard age-structured model, for years $y = 1975, \dots, 2021$ and for ages $a = 1, \dots, 20+$, where $20+$ represents the plus group.

$$\begin{aligned}\log(N_{y,a}) &= \log(N_{y-1,a-1}) - Z_{y-1,a-1} + \delta_{N_{y-1,a-1}}, \\ \log(N_{y,A^+}) &= \log[N_{y-1,A^+-1} \exp^{-Z_{y-1,A^+-1}} + N_{y-1,A^+} \exp^{-Z_{y-1,A^+}}] + \delta_{N_{y-1,A^+}},\end{aligned}$$

where the process errors are assumed independent $\delta_{N_{y,a}} \sim N(0, \sigma_{pe})$, and $Z_{y,a} = M_{y,a} + F_{y,a}$ is the total mortality rate given by the sum of the natural mortality rate, $M_{y,a}$, and the fishing mortality rate $F_{y,a}$. Here, $M_{y,a}$ were predicted as a function of body weight using the Lorenzen method,

$$M_a = M_o^{-0.305},$$

where W_a is the stock weight-at-age, M_o is a scaling parameter and -0.305 is from Lorenzen (1996). Here, $M_o = \mu(M_a / (\min M_a))$, where $\mu = 0.125$ is the median natural mortality rate from Cadigan et al. (2022). Weight-at-age was derived using a stock length-weight relationship and a length-age relationship,

$$W_a = \alpha L_a^\beta,$$

where L_a are internally estimated using the von Bertalanffy growth model (detailed below). The length-weight parameters α and β were derived from length-weight estimates from the EU-Spain research vessel surveys. Recruitment, i.e., the numbers at the first ages $N_{y,1}$, are treated as random deviations from a fixed mean effect,

$$\log(N_{y,1}) = \mu_R + \delta_{R_y},$$

where μ_R is the mean recruitment, split for years pre/post 2005 to account for the large cohort that appeared around that time. The deviations from the mean recruitment δ_{R_y} are assumed to follow a normal distribution with AR(1) correlation across years with the AR parameters σ_R^2 and ϕ_R to be estimated.

The ACL model does not have direct information about the initial age-distribution of the stock, therefore a prior based on the numbers in the first year is given by $\log\left(\frac{N_{a,1}}{N_{a-1,1}}\right) \sim iidN(-Z_{a-1,1}, \sigma_{N_0}^2)$, with $\sigma_{N_0}^2$ fixed at 0.2. Fixing $\sigma_{N_0}^2$ at 0.2 allows enough flexibility for the initial age distribution of the stock to deviate from the prior, but not enough to have wildly varying numbers in the first year.

The fishing mortality rates are set to zero for ages 1-2, as it is assumed that the fishing gear does not target those smaller sizes, and for all ages greater than two,

$$\log(F_{y,a}) = \mu_{F_{y,a}} + \delta_{F_{y,a}},$$

where $\mu_{F_{y,a}}$ is the mean fishing mortality rate and $\delta_{F_{y,a}}$ is the deviation from the mean at each age and year. A separate $\mu_{F_{y,a}}$ is estimated for for years 1998-2009 in order to account for the change in

fishing mortality rates during the moratorium. The $\delta_{F_{y,a}}$'s are treated as random effects and are assumed to follow a normal distribution with parameters $\sigma_F^2, \phi_{F_A}, \phi_{F_Y}$ to be estimated.

Probability of length at age

Consistent ageing data are not available for 3LN redbfish, therefore, the numbers at age in the model need to be converted to numbers at length. Starting with the modified von Bertalanffy growth model from Cadigan and Campana ([Cadigan and Campana, 2017](#)), we define the mean length at age L_a as

$$L_a = L_\infty(1 - e^{-Ka}(1 - \rho_0)),$$

where L_∞ is the theoretical maximum average length, $p_0 = \frac{L_0}{L_\infty}$, L_0 is the mean length at birth, and K is the growth rate parameter. The length information is given for one centimeter length bins, and we assume that there is a mid-point of the length bins. Therefore, a fish of length bin l will have length $L \in (l - 0.5\text{cm}, l + 0.5\text{cm})$. The cumulative distribution function is used to calculate the probability that a fish is in length bin l given its age. As is often the case in fisheries applications, we assume that length-at-age is normally distributed with mean L_a , with standard deviation that increases with the mean, i.e., $L_a \sim N(L_a, \tau L_a)$. Then, the CDF for the normal distribution give the probability that a fish is in length bin l given its age,

$$P(L_a \in l) = \Phi\left(\frac{l - L_a + 0.5}{\tau L_a}\right) - \Phi\left(\frac{l - L_a - 0.5}{\tau L_a}\right)$$

The number of fish in each length bin (N_{ly}) is then given by

$$N_{ly} = \sum_a N_{ay} P_{la}$$

where P_{la} is shorthand for $P(L_a \in l)$.

Observation model

The Baranov catch equation relates the expected catch to the stock size,

$$C_{y,a} = \frac{F_{y,a}}{Z_{y,a}} (1 - \exp^{-Z_{y,a}}) N_{y,a}.$$

where we convert catch at age to catch at length using the approach described above,

$$C_{ly} = \sum_a C_{ay} P_{cla},$$

with P_{cla} representing the assumption that the catch are sampled in the middle of the year (i.e. we add 0.5 to the ages). To account for uncertainties in the observed catch, we add an observational error term to describe the relationship between the predicted and observed catch (C_{oly}),

$$C_{oly} = C_{ly} e^{\epsilon_{cty}}$$

where $\epsilon_{c,l_1,\dots,l_L,y} \sim iid\ MVN(0, \Sigma_C)$, i.e. observed catch are assumed correlated across lengths within years, but otherwise independent within years. Preliminary model exploration indicated that this

assumption is important. Similarly, the relationship between stock size and the timing of the survey is given by

$$N_{say} = N_{ay} e^{-f_s Z_{ay}},$$

where f_s represents the fraction of the year that the survey takes places. The number of fish in each length bin is given by

$$N_{sly} = \sum_a N_{say} P_{sla},$$

with P_{sla} representing the probability of length at age for each survey $P_{sla} = p_{l,(a+f_s)}$. The expected survey index is then

$$E_{Isly} = q_{gsl} N_{sly},$$

where q_{gsl} represents a separate catchability at length estimate for each survey and gear. Catchability is assumed from a random walk,

$$q_{sl+1} = q_{sl} e^{\epsilon_{ql}}$$

with $\epsilon_{ql} \sim N(0, \sigma_{q,g,s,l})$. To avoid unrealistically fluctuating catchability estimates at the smallest sizes, $\sigma_{q,g,s,l}$ was fixed at 0.20 for lengths less than 16cm and greater than 31cm. Estimates of q_{sl} for lengths greater than 45cm were fixed at the estimate of 45cm. To avoid confounding between q_{sl} and N_{sly} , we added a medium prior on the Fall RV q_{sl} , where we assumed that q_{sl} was approximately 1 for lengths 20, 22 and 24.

To account for uncertainties in the observed indices, we add an observational error term to describe the relationship between the predicted and observed index

$$I_{sly} = E_{Isly} e^{\epsilon_{sly}}$$

The ϵ_{sly} sampling errors were multivariate normal (MVN) with mean zero and lag-1 autoregressive correlation ρ_l among length classes each year, and conditional standard deviations σ_{sl} that were estimated separately for each survey s . For simplicity the autocorrelation was assumed to be the same for all surveys.

Colophon

This version of the document was generated on 2023-06-06 10:35:28 using the R markdown template for SCR documents from [NAFOdown](#).

The computational environment that was used to generate this version is as follows:

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#> setting value
#> version R version 4.2.2 (2022-10-31 ucrt)
#> os Windows 10 x64 (build 19044)
#> system x86_64, mingw32
#> ui RTerm
#> language (EN)
#> collate English_United States.utf8
#> ctype English_United States.utf8
#> tz America/St_Johns
#> date 2023-06-06
#> pandoc 2.19.2 @ C:/Program Files/RStudio/bin/quarto/bin/tools/ (
via rmarkdown)
#>
#> - Packages -----
#> package * version date (UTC) lib source
#> askpass 1.1 2019-01-13 [1] CRAN (R 4.2.2)
#> assertthat 0.2.1 2019-03-21 [1] CRAN (R 4.2.2)
#> base64enc 0.1-3 2015-07-28 [1] CRAN (R 4.2.0)
#> bookdown 0.31 2022-12-13 [1] CRAN (R 4.2.2)
#> cachem 1.0.6 2021-08-19 [1] CRAN (R 4.2.2)
#> callr 3.7.3 2022-11-02 [1] CRAN (R 4.2.2)
#> cellranger 1.1.0 2016-07-27 [1] CRAN (R 4.2.2)
#> cli 3.4.1 2022-09-23 [1] CRAN (R 4.2.2)
#> colorspace 2.0-3 2022-02-21 [1] CRAN (R 4.2.2)
#> crayon 1.5.2 2022-09-29 [1] CRAN (R 4.2.2)
#> data.table 1.14.4 2022-10-17 [1] CRAN (R 4.2.2)
#> DBI 1.1.3 2022-06-18 [1] CRAN (R 4.2.2)
#> devtools 2.4.5 2022-10-11 [1] CRAN (R 4.2.2)
#> digest 0.6.30 2022-10-18 [1] CRAN (R 4.2.2)
#> dplyr 1.0.10 2022-09-01 [1] CRAN (R 4.2.2)
#> ellipsis 0.3.2 2021-04-29 [1] CRAN (R 4.2.2)
#> evaluate 0.18 2022-11-07 [1] CRAN (R 4.2.2)
#> fansi 1.0.3 2022-03-24 [1] CRAN (R 4.2.2)
#> fastmap 1.1.0 2021-01-25 [1] CRAN (R 4.2.2)
#> flextable * 0.8.3 2022-11-06 [1] CRAN (R 4.2.2)
#> fs 1.5.2 2021-12-08 [1] CRAN (R 4.2.2)
#> gdtools 0.2.4 2022-02-14 [1] CRAN (R 4.2.2)
#> generics 0.1.3 2022-07-05 [1] CRAN (R 4.2.2)
#> ggplot2 * 3.4.0 2022-11-04 [1] CRAN (R 4.2.2)
#> ggribes 0.5.4 2022-09-26 [1] CRAN (R 4.2.2)
#> ggthemes 4.2.4 2021-01-20 [1] CRAN (R 4.2.2)
#> glue 1.6.2 2022-02-24 [1] CRAN (R 4.2.2)
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wn@f9eda23)
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#> xfun           0.35       2022-11-16 [1] CRAN (R 4.2.2)
#> xml2           1.3.3      2021-11-30 [1] CRAN (R 4.2.2)

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#>
#> [1] C:/Users/perreaultan/AppData/Local/R/win-library/4.2
#> [2] C:/Program Files/R/R-4.2/library
#>
#> _____
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