Northwest Atlantic Fisheries Organization

Serial No. N7522 **NAFO SCR Doc. 24/017**

SCIENTIFIC COUNCIL MEETING – JUNE 2024

Analysis of Stock-Recruitment Relationships for NAFO Div. 3M Cod in Preparation for MSE Simulations

Nick Gullage[1], Rajeev Kumar[1], Divya Varkey[1]

[1]Fisheries and Oceans Canada, St. John's, CA
2024-06-03

Introduction

We were tasked with investigating Stock-Recruit various relationships (SRRs) for NAFO Div. 3M cod to be used in the MSE simulations for stock projections. Here we model and review the fitness of Beverton-Holt and Ricker models with different *steepness* parametrizations.

Methods

Steepness is "the proportion of equilibrium unexploited recruitment produced by 20% of unexploited spawning stock size" (**Miller and Brooks, 2021**), and is used to reparametrize SRR with respect to virgin spawning biomass and recruitment, S_0 and R_0, respectively. The Beverton-Holt (BH) model is defined as

$$R(S) = \frac{4R_0hS}{(1-h)R_0\phi_0 + (5h-1)S}$$

where h is the steepness parameter, and $\phi_0 = SPR(0)$ is spawners-per-recruit (SPR), defined here as a function of F, evaluated at $F = 0$ for a given posterior parameter set (see Appendix A). The Ricker (RK) model is defined as

$$R(S) = \frac{S}{\phi_0}(5h)^{\frac{5}{4}\left(1-\frac{S}{R_0\phi_0}\right)}$$

Recruitment model were fitted with fixed values of h, and $S_0 = SPR(0) * R_0$, where R_0 was estimated given posterior stock-recruit data (i.e. SSB and recruit abundance) from the assessment outputs for 2022. We used two method for fitting Stock-recruit curves:

1) Stock-recruit data for each posterior output ($n = 1000$) were fit individually to stock-recruit functions, and R_0 was derived as the median estimate of all fits.

2) Stock-recruit data was summarized as medians of posterior outputs, the median data were fit to the stock-recruit functions, and a single value for R_0 was derived.

SPR was calculated for each posterior data set for *method (1)* which was used to define ϕ_0 for each independent stock-recruit function. However, for *method (2)*, and when plotting median trajectories of Recruits vs. SSB, median SPR was used, defined using the median of posterior LHC values within the SPR function. Lastly, 90% confidence intervals (CIs) were calculated for each stock-recruit model using the the posterior outputs of stock-recruit data. CIs are defined as the 95^{th} and 5^{th} quantiles of R_0 using posterior data as in *method (1)*, which were then used to project recruits given SSB for both high and low estimates of R_0.

Stock-recruit models were fit assuming constant values of steepness, where h = 0.9, 0.75, 0.7, 0.65, & 0.5 for both the BH and RK models, giving 10 different model fits (Figure 1 & 2). Root-mean-square-error (RMSE) and AIC for each model were calculated to determine goodness of fit. Maximum Sustainable Yield (MSY) reference points (RPs) were derived using a standard equilibrium yield analysis (see Appendix B). RPs required for the precautionary approach (PA) leaf harvest control rule (HCR), i.e. B_{lim}, $B_{trigger}$, and F_{target}, were also derived for each model to supplement upcoming projection work. Additionally, RPs required for the leaf HCR were used to plot the PA status for each respective fitted SRR model and also to compare relative benchmarks against the previous estimates of $B_{lim} = 14564$ and $SSB_{2022} = 29545$.

A separate SRR was fit for each of the BH and RK models by estimating both R_0 and h, simultaneously. AIC, RMSE, and RP values were derived for both models, as well.

Results & Discussion

Model fits from both *method (1)* and *(2)* were indistinguishable. Results, and future fits to the SRRs, will use method *(2)* because it is less computationally demanding.

All Ricker models fit the data better than then all Beverton-Holt models, according to AIC and RMSE values (Table 1), with RK h = 0.9 being the best fit model. Confidence intervals on the Beverton-Holt models are tighter than those for the Ricker models, which become unreliably large for greater values of SSB (Figures 5-13). When considering projections, caution is warranted in SRR selection because the fitted Ricker models may provide highly variable results in simulations, despite these models better fitting the data.

MSY RPs for the Ricker models provided more higher BRPs, FRPs and MSY compared to the Beverton-Holt model (Table 3). For the RK models, higher steepness values resulted in strictly higher estimates for BRPs and FRPs. For the BH models a moderate steepness (i.e. h = 0.75) provided the highest estimate for BRPs and FRPs, but steepness values on either extreme (i.e. h = 0.9 & h = 0.5) provided notably lower estimates for BRPs and FRPs, with h = 0.5 providing the lowest values.

The SRRs that are considered the best fits (i.e. RK with h = 0.9, 0.75, & 0.7) also indicate SSB_{2022} to be farther into the critical zone and $B_{trigger}$ to be farther into the cautious zone than the original leaf HCR (Figure 15), and some of the worst fits (e.g. BH with h = 0.9 & 0.5) show the opposite trend with the original $B_{trigger}$ far exceeding the estimated $B_{trigger}$ for their respective models. Furthermore, the better fitting SRRs suggest that more precaution (i.e. fewer catches) may be required to achieve a healthy status due to $B_{trigger}$ being much larger than the original $B_{trigger} = 36366$ and other estimates of it. Lastly, better fitting SRR models have higher values of F and

F_{target}, indicating that, despite requiring higher levels of SSB to recover, they would provide more catches once in the healthy zone and overall.

Model fits for the BH and RK models by estimating both R_0 and h were also included (Figure 3 & Figure 4). AICs were among the lowest and RMSEs were the lowest for their respective stock-recruit models (Table 2), and fitted curves were similar to the other models, as well. Estimated values for the optimal h were within the range of previously tested values for the BH model, but far exceeded the tested range of tested values for the RK model. RPs for each optimal model are similar to previous estimates of each respective model (Table 4), and leaf HCRs also show similarity with previous models (Figure 16).

Overall, all SRRs show similar shapes and parameter estimates, and AICs for most models tested were comparable, with the optimal fits have lowest values for RMSE and AIC for respective SRRs. Lower values of steepness lead to higher estimates for B_{MSY} and lower estimates for F_{MSY}. The optimal RK had the lowest B_{lim} (of the RK SRRs) and highest MSY & F_{target}, while optimal BH had the second lowest B_{lim} (of the BH SRRs), the highest MSY, and the second highest F_{target}. Our recommendation is to select the two optimal SRRs for implementation in the MSE model.

Tables

Table 1. RMSE and AIC for each model fit (bolded values indicate the model of best fit).

SRR	h	log(R_0)	Std (logR_0)	RMSE	AIC
BH	0.90	11.76	0.18	93,221	879
BH	0.75	11.92	0.20	90,709	877
BH	0.70	11.96	0.22	91,153	877
BH	0.65	12.01	0.25	91,990	878
BH	0.50	12.15	0.39	96,385	881
RK	**0.90**	**11.57**	**0.15**	**87,344**	**874**
RK	0.75	11.63	0.19	90,082	876
RK	0.70	11.66	0.21	91,262	877
RK	0.65	11.69	0.24	92,584	878
RK	0.50	11.84	0.38	97,448	882

Table 2. RMSE and AIC for each model fit based on the optimal h.

SRR	h	$\log(R_0)$	Std (h)	Std ($\log R_0$)	RMSE	AIC
BH	0.772	11.90	0.149	0.26	90,660	879
RK	1.230	11.47	0.289	0.15	85,078	874

Table 3. MSY Reference Points for each model fit.

SRR	h	F_{MSY}	B_{MSY}	MSY	B_{lim}	$B_{trigger}$	F_{target}
BH	0.90	0.328	35,886	13,553	10,766	26,915	0.278
BH	0.75	0.253	48,326	14,179	14,498	36,244	0.215
BH	0.70	0.232	52,214	14,083	15,664	39,161	0.197
BH	0.65	0.212	56,252	13,890	16,876	42,189	0.180
BH	0.50	0.152	71,633	12,859	21,490	53,724	0.129
RK	0.90	0.289	44,682	14,923	13,405	33,512	0.246
RK	0.75	0.242	48,708	13,675	14,612	36,531	0.205
RK	0.70	0.225	50,400	13,214	15,120	37,800	0.191
RK	0.65	0.208	52,379	12,731	15,714	39,285	0.177
RK	0.50	0.153	61,928	11,192	18,579	46,446	0.130

Table 4. MSY Reference Points for each model fit based on the optimal h.

SRR	h	F_{MSY}	B_{MSY}	MSY	B_{lim}	$B_{trigger}$	F_{target}
BH	0.772	0.262	46,652	14,187	13,996	34,989	0.223
RK	1.230	0.387	38,415	17,081	11,524	28,811	0.329

Figures

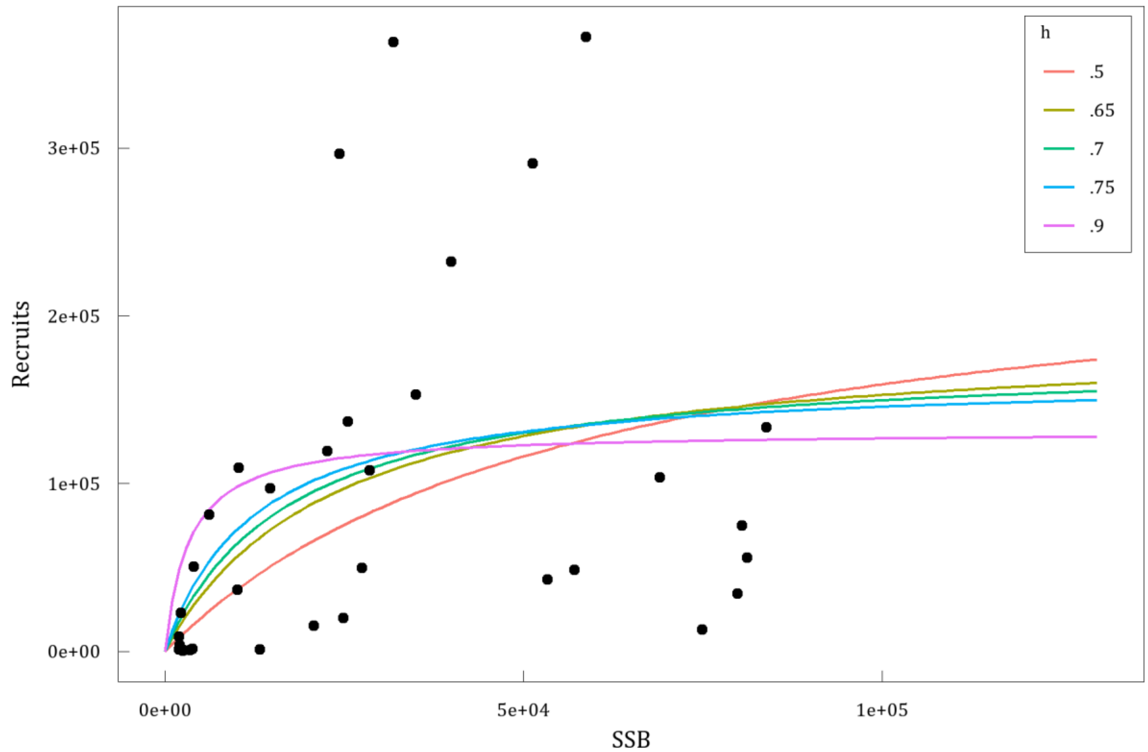

Figure 1. Model fits for all fixed steepness values for the Beverton-Holt SRR.

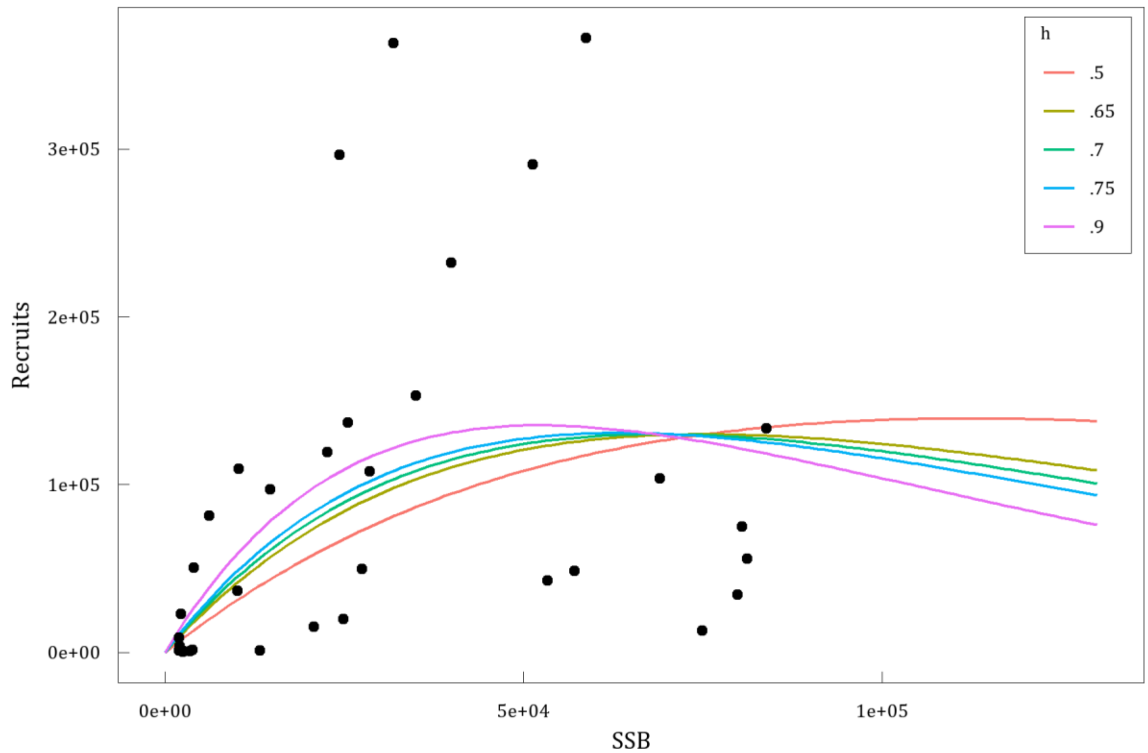

Figure 2. Model fits for all fixed steepness values for the Ricker SRR.

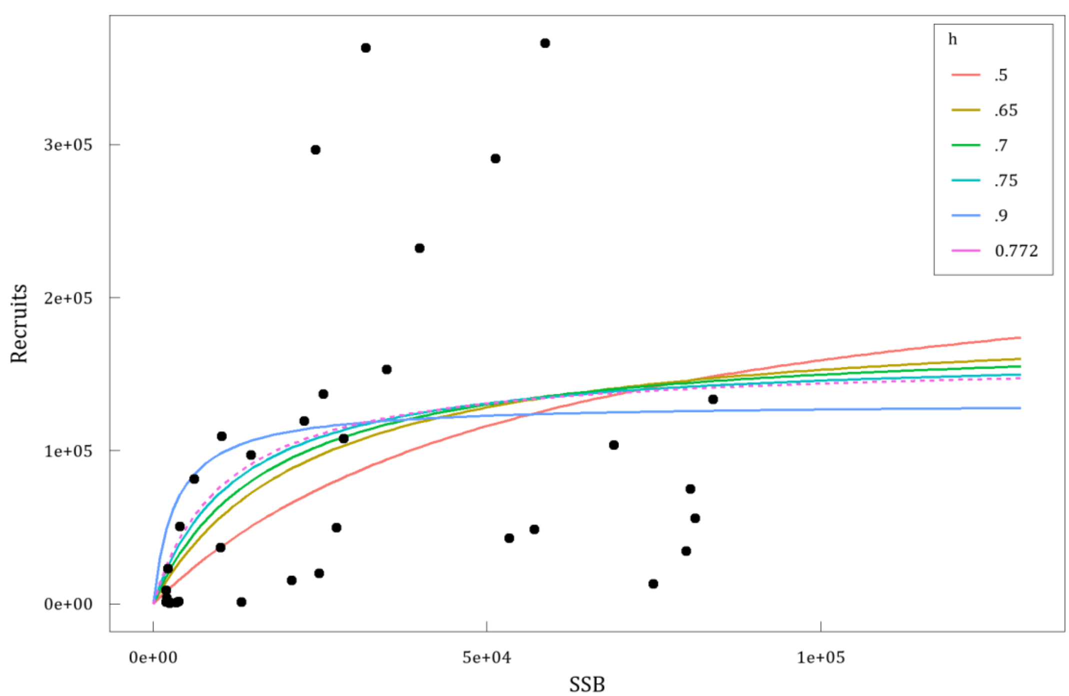

Figure 3. Model fits for all fixed steepness values (solid) compared to the optimal steepness (dashed) for the Beverton-Holt SRR.

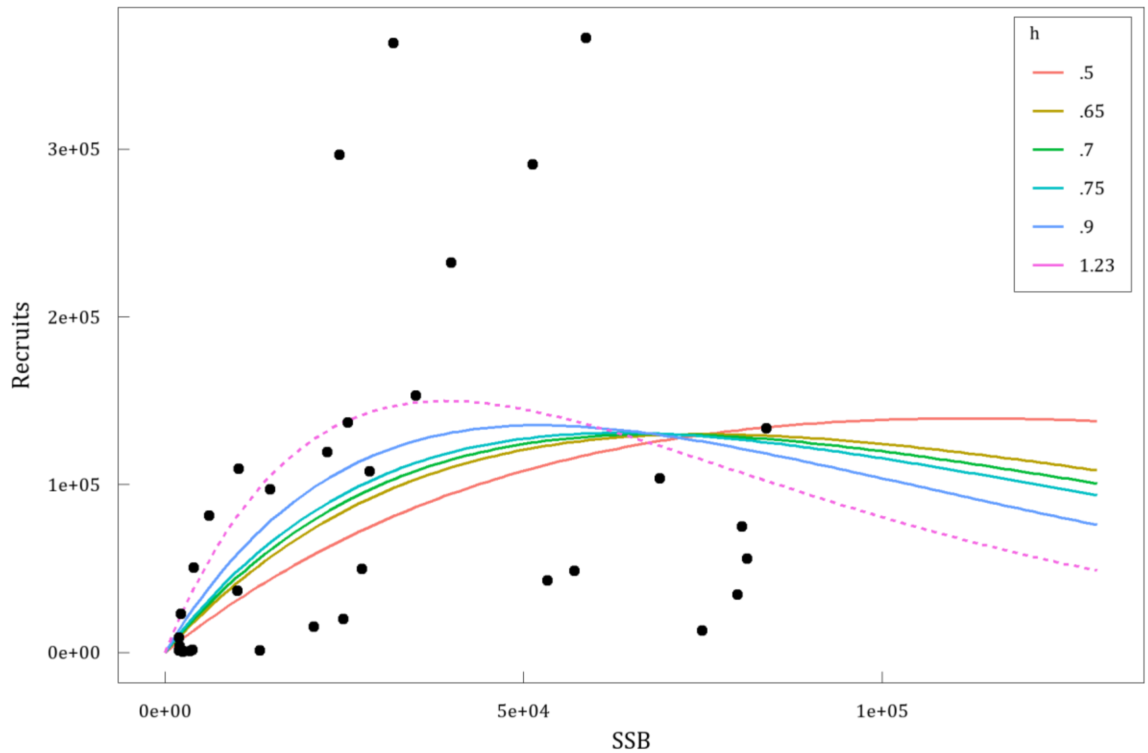

Figure 4. Model fits for all fixed steepness values (solid) compared to the optimal steepness (dashed) for the Ricker SRR.

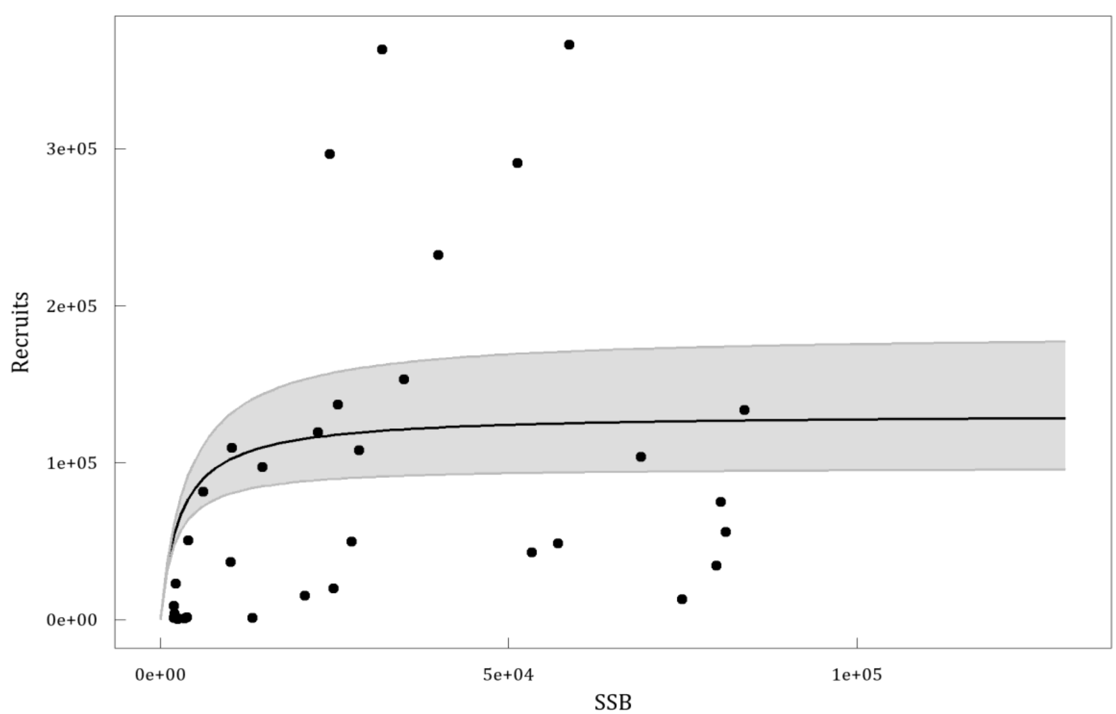

Figure 5. Median and 90% CIs for the BH SRR with h = 0.9.

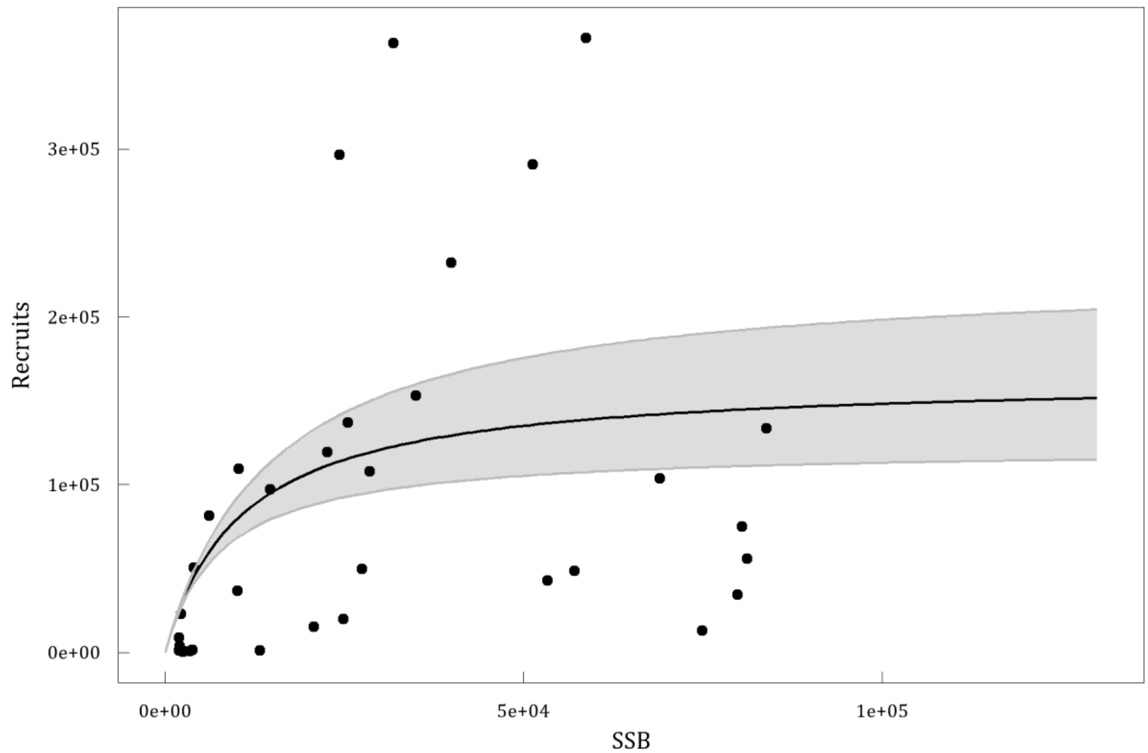

Figure 6. Median and 90% CIs for the BH SRR with h = 0.75.

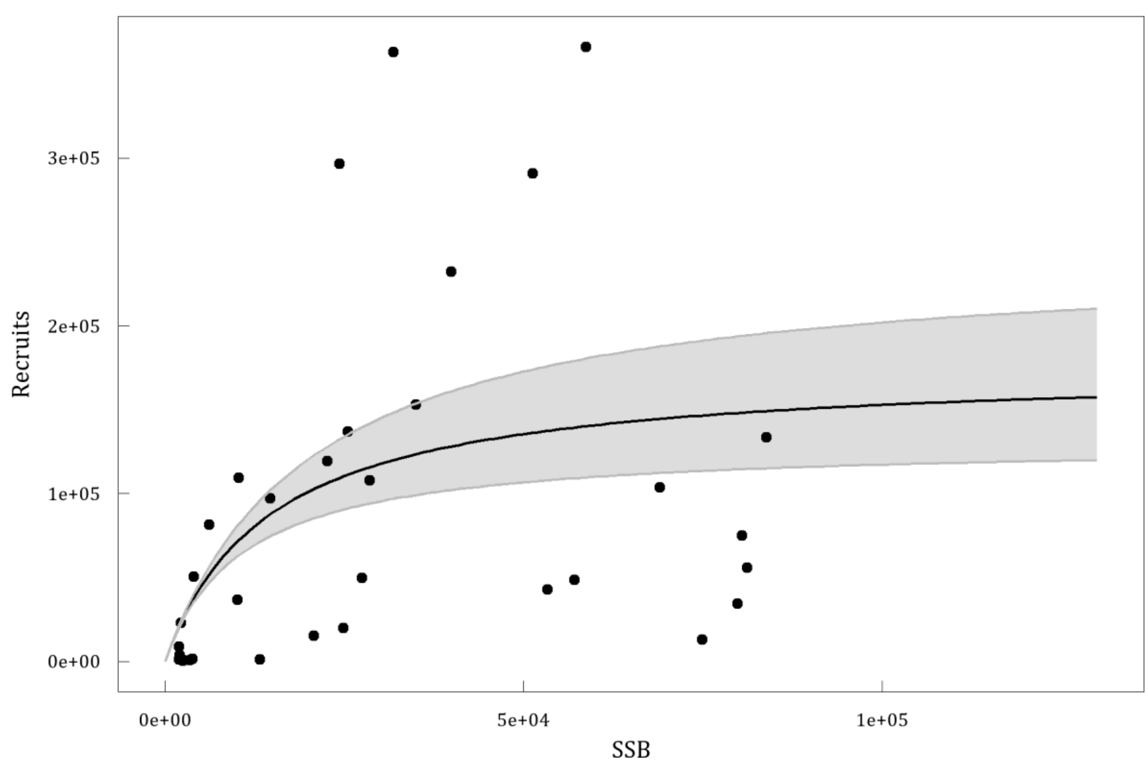

Figure 7. Median and 90% CIs for the BH SRR with h = 0.7.

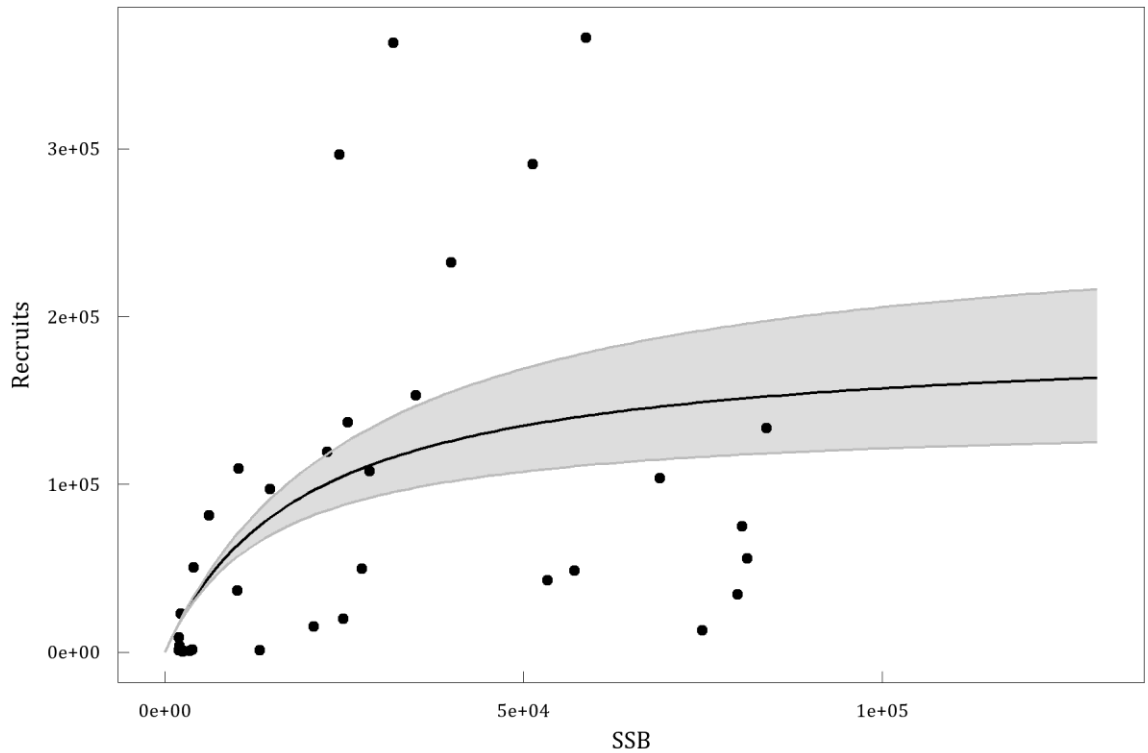

Figure 8. Median and 90% CIs for the BH SRR with h = 0.65.

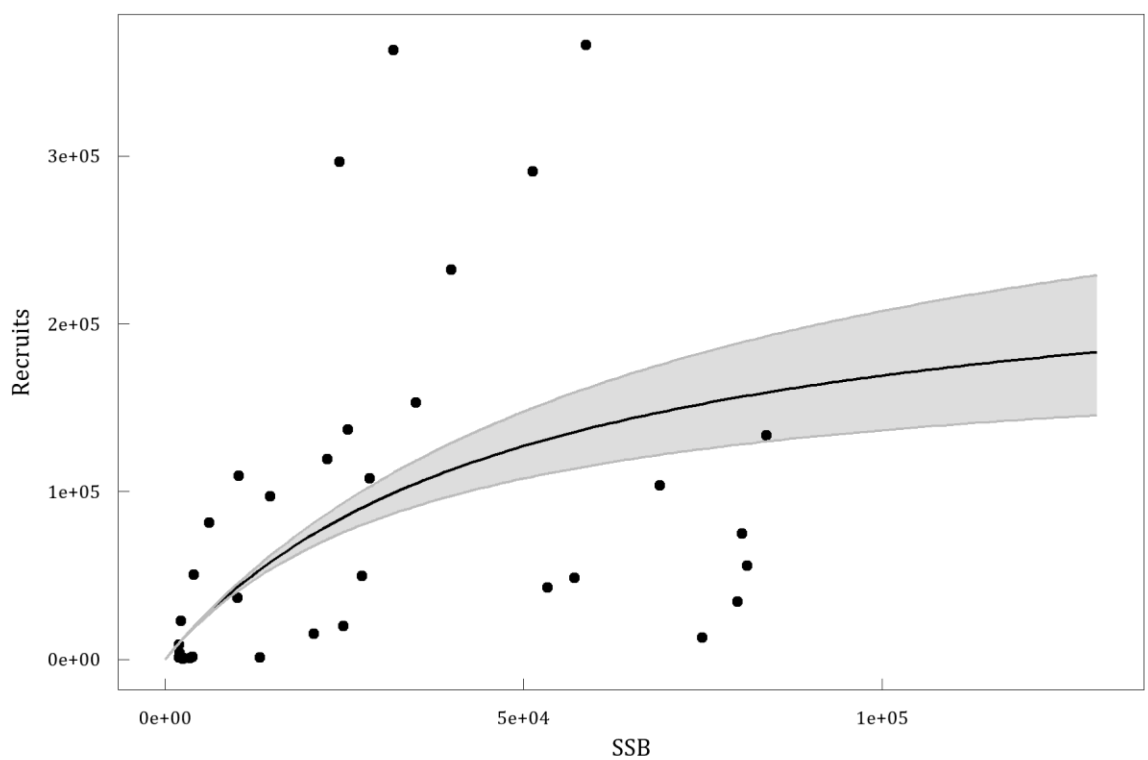

Figure 9. Median and 90% CIs for the BH SRR with h = 0.5.

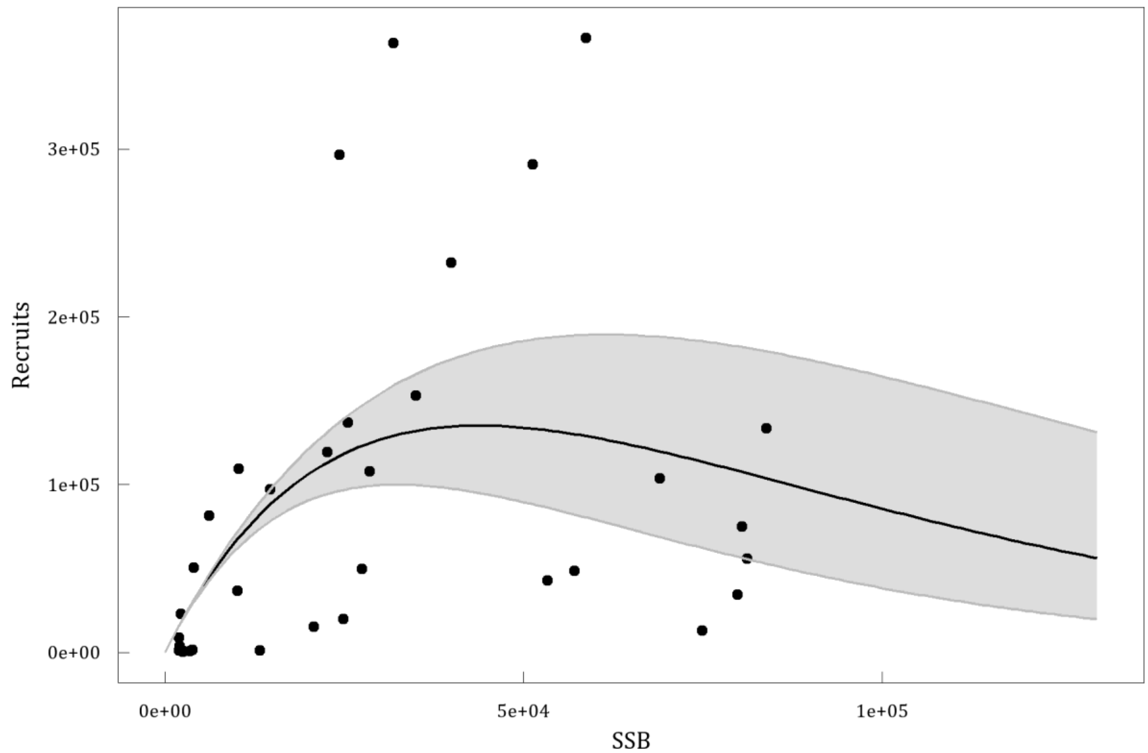

Figure 10. Median and 90% CIs for the RK SRR with h = 0.9.

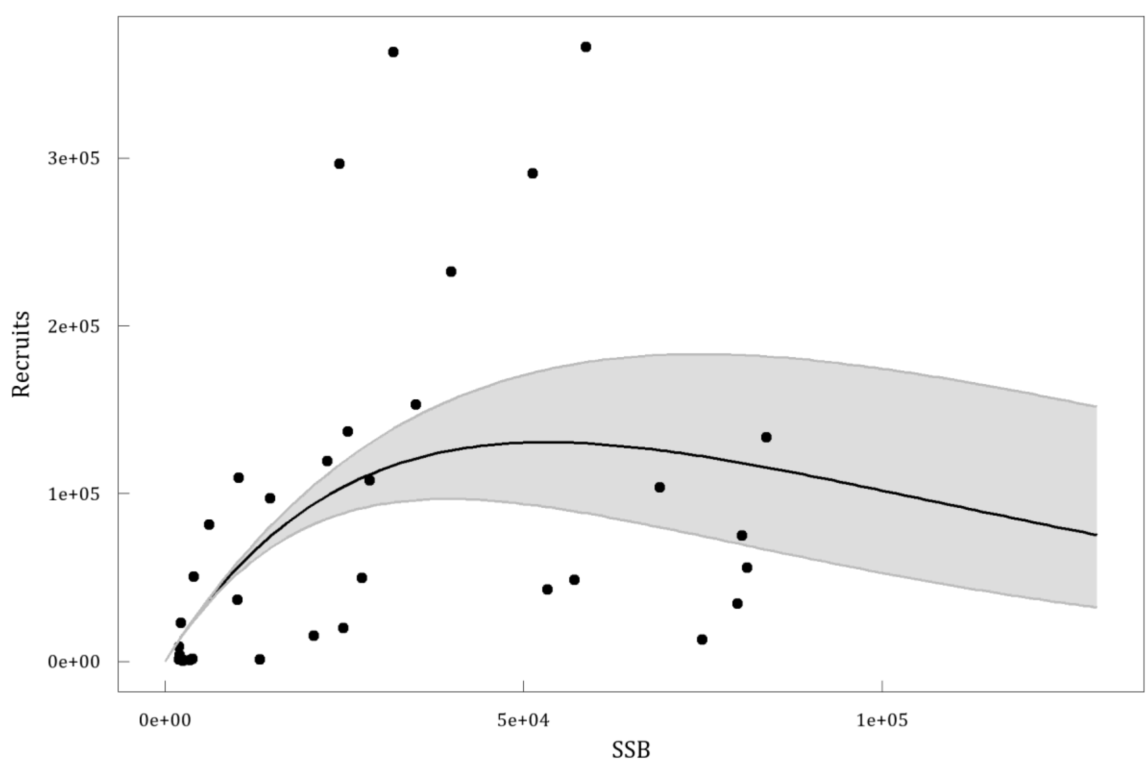

Figure 11. Median and 90% CIs for the RK SRR with h = 0.75.

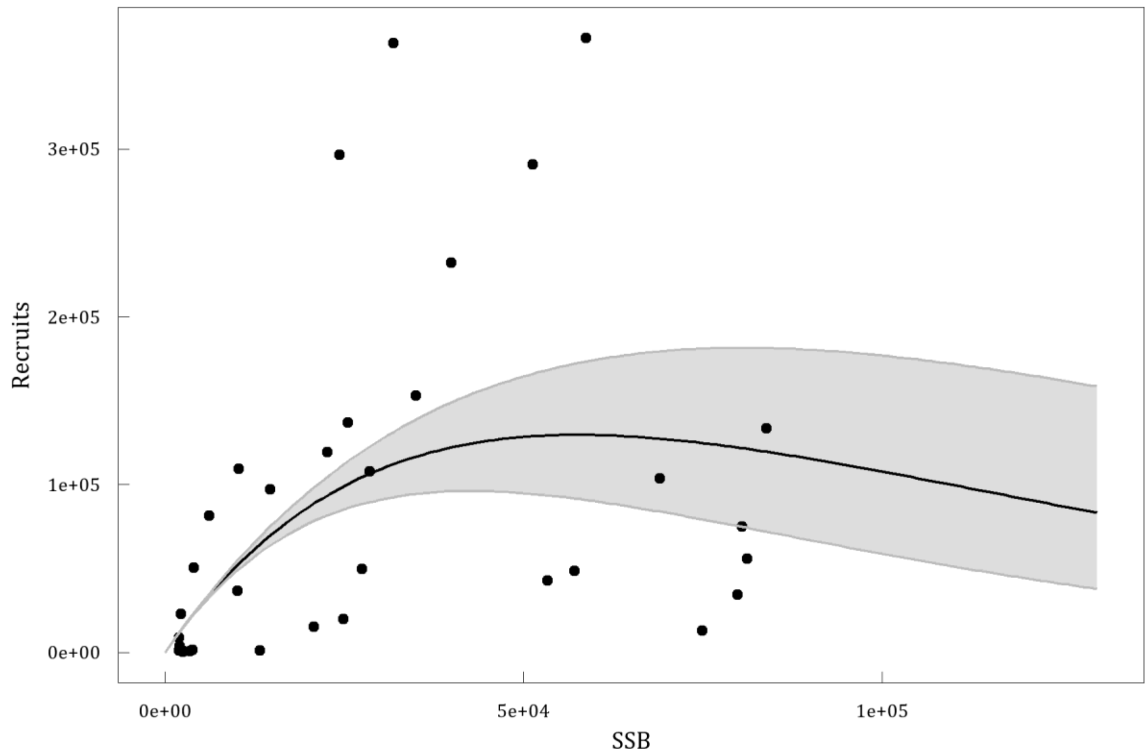

Figure 12. Median and 90% CIs for the RK SRR with h = 0.7.

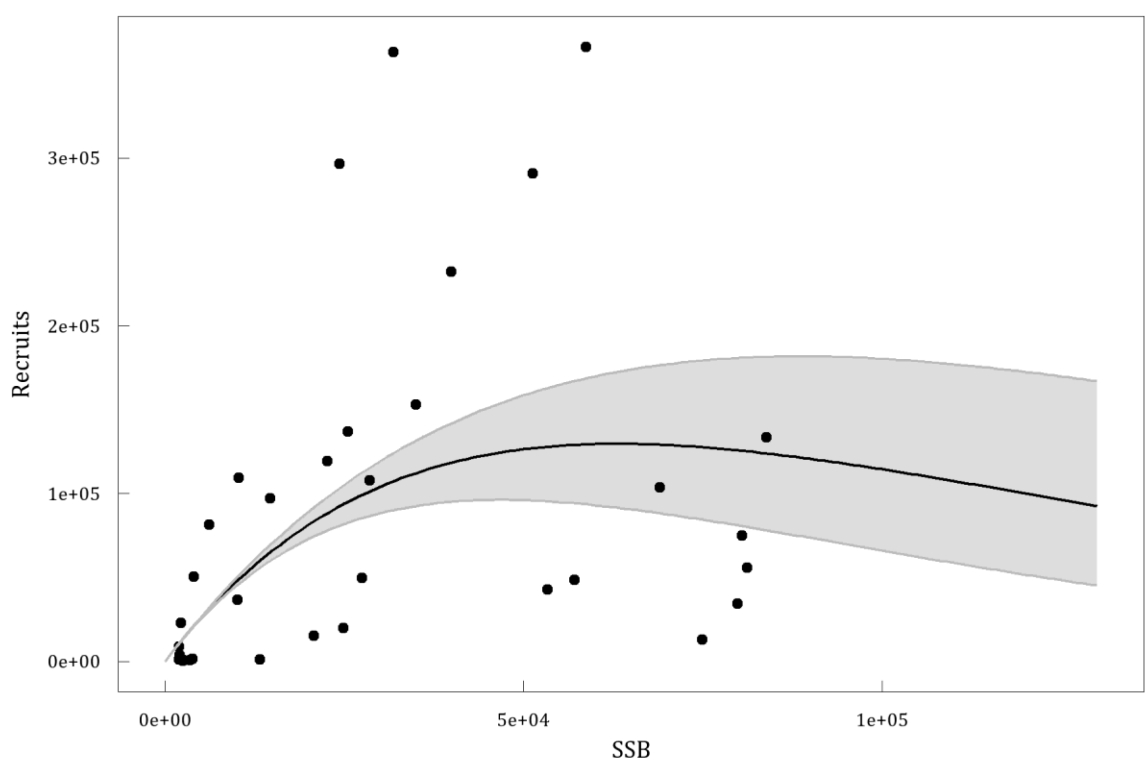

Figure 13. Median and 90% CIs for the RK SRR with h = 0.65.

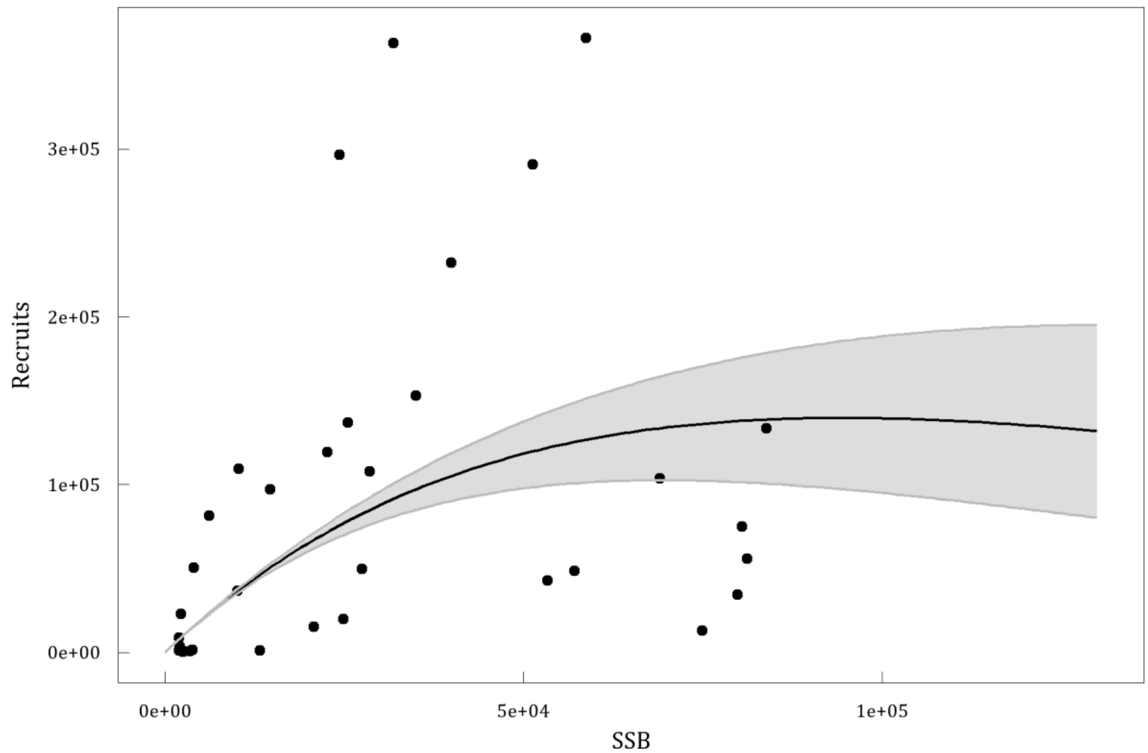

Figure 14. Median and 90% CIs for the RK SRR with h = 0.5.

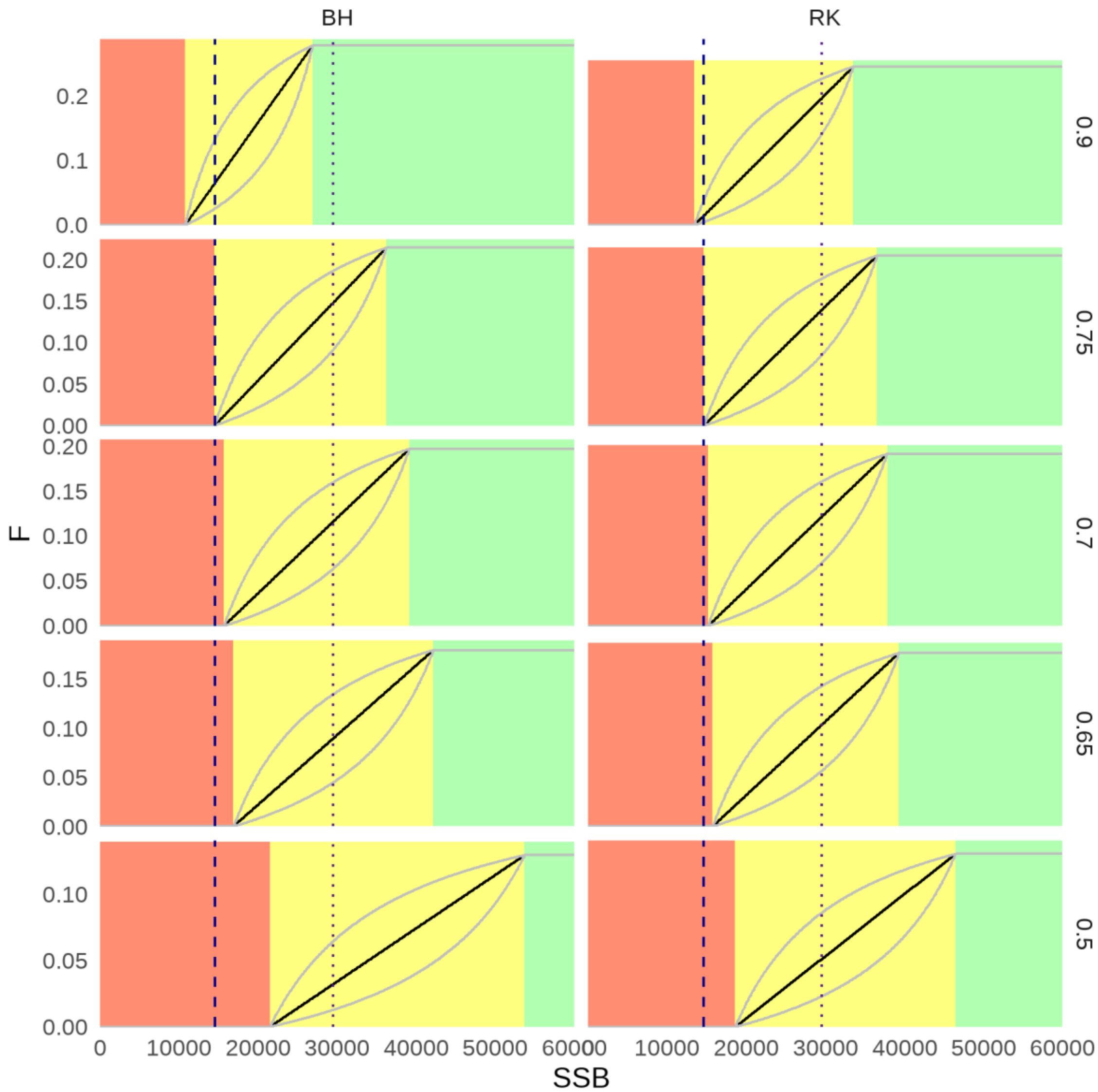

Figure 15. Precautionary Approach harvest control rule leaves based on reference points (i.e. B_{lim}, $B_{trigger}$, and F_{target}) for each stock-recruit curve fit (SRR by column, steepness by row). The dashed (blue) vertical line is the original B_{lim} = 14564, and the dotted (purple) vertical line is SSB_{2022} = 29545.

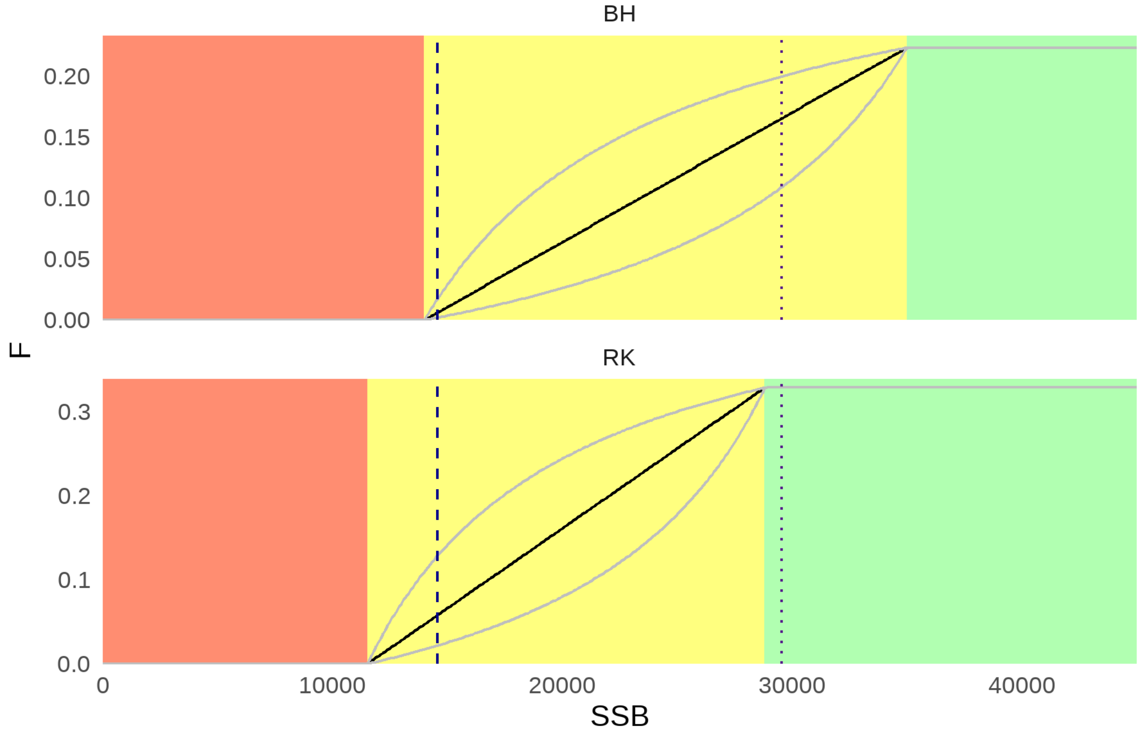

Figure 16. Precautionary Approach harvest control rule leaves based on reference points (i.e. B_{lim}, $B_{trigger}$, and F_{target}) for both of the optimal stock-recruit curve fits. The dashed (blue) vertical line is the original B_{lim} = 14564, and the dotted (purple) vertical line is SSB_{2022} = 29545.

References

Miller, T. J., and Brooks, E. N. (2021). Steepness is a slippery slope. *Fish and Fisheries*, *22*(3), 634–645.

Appendix A

The spawner-per-recruit is defined as a function which evaluates the ratio of spawning biomass to recruits for a given level of fishing mortality, where

$$SPR(f) = \sum_a M_a \, W_a^b e^{-\Sigma_j^{a-1} Z_j},$$

where M_a is maturity-at-age, W_a^b is the beginning-of-year (i.e. stock) weight-at-age, and the exponentiated term is the cumulative sum of total mortalities up to a given age, as indicated by a standard cohort equation, which gives a ratio of abundance-at-age to recruit abundance. here, Z_j is a function of natural, M_a, and fishing, F_a, mortality where fishing mortality is assumed to be a function of selectivity, $F_a = f \cdot sel_a$, and f is constant.

For calculating recruitment trajectories and estimating R_0 for the median fit (*method (2)*), we derive SPR and YPR using the median values across posterior samples, and years (i.e. using the entire timeseries), for maturities-at-age and natural mortalities-at-age. Values for selectivity and weights-at-age are constant across posterior samples, and are applied as median across years, only.

Appendix B

Maximum sustainable yield (MSY) reference points (RPs) are defined as the global optimum of an equilibrium yield curve with respect to *f*. Equilibrium Yield is defined as the long-term stable state of yield from a population that is fished at a constant harvest rate, defined as

$$Y_{eq}(f) = YPR(f) R_{eq},$$

where R_{eq} is the recruitment produced by a stock-recruit relationship for every year at equilibrium, and YPR is the yield-per-recruit.

The yield-per-recruit is defined as a function which evaluates the ratio of yield to recruits for a given level of fishing mortality, where

$$YPR(f) = \sum_a P_a \, W_a^m e^{-\Sigma_j^{a-1} Z_j},$$

where W_a^m is the mid-year (i.e. catch) wight-at-age, and

$$P_a = \frac{F_a}{Z_a}\big(1 - exp(-Z_a)\big),$$

is the proportion of catches take for age group a (as in the Baranov catch equation).

At equilibrium, we assume that the age structure of a population across years is equivalent to the age structure of a population within a yearfor example, at equilibrium the abundance of age 3 fish for one year is equivalent to the abundance of age 3 fish for the next year and every subsequent year. This is defined as follows,

$$S_{eq} = SPR(f)R_{eq},$$

where S_{eq} is the SSB for every year at equilibrium. Recruitment is assumed to be function of SSB, and so we can say

$$S_{eq} = SPR(f)R(S_{eq}),$$

and equilibrium SSB can derived by solving the above equation for S_{eq} for a given stock-recruit relationship, where S_{eq} is a function of *f*. Equations for equilibrium SSB are not derived explicitly for the stock-recruit functions used herein, and values are instead solved for numerically using `uniroot` in R for each parametrization. Equilibrium recruitment can be defined as a function of *f* through the equation for equilibrium SSB, such that $R_{eq}(f) = R\left(S_{eq}(f)\right)$. Equilibrium yield can then be optimized with respect to *f*. The value of *f* which optimizes Y_{eq} is F_{MSY}. MSY is the equilibrium yield at F_{MSY}, and B_{MSY} is the equilibrium SSB at F_{MSY}. Equilibrium yield functions are optimized in R using the `nlminb` function (where the objective to be minimized is defined as $-Y_{eq}^2$).