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Re-estimation of equilibrium reference points for NAFO Division 3M Cod for application in MSE simulations.

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Introduction

Estimates for reference points (RPs) for NAFO Div. 3M cod are required for the precautionary approach (PA) leaf harvest control rules (HCRs). RPs were derived to be used within a management strategy evaluation (MSE, see **Kumar et al., 2024**). Previously, RPs were derived using the average of LHCs over the entire time series of the stock (**Gullage et al., 2024**). However, various change and suggestion required RPs to be recalculated so that estimates are consistent with recent stock trends. Here we fit a Beverton-Holt (BH) and Ricker (RK) stock-recruit relationship (SRR) with different *steepness* parametrizations and different sets of life history characteristics, and derive various RPs for each fit model.

Methods

Steepness is "the proportion of equilibrium unexploited recruitment produced by 20% of unexploited spawning stock size" (**Miller and Brooks, 2021**), and is used to reparametrize SRR with respect to virgin spawning biomass and recruitment, S_0 and R_0, respectively. The BH model is defined as

$$R(S) = \frac{4R_0hS}{(1-h)R_0\phi_0 + (5h-1)S}$$

where h is the steepness parameter, and $\phi_0 = SPR(0)$ is spawners-per-recruit (SPR), defined here as a function of F, evaluated at $F = 0$ for a given posterior parameter set (see Appendix A). The Ricker (RK) model is defined as

$$R(S) = \frac{S}{\phi_0}(5h)^{\frac{5}{4}\left(1-\frac{S}{R_0\phi_0}\right)}$$

Recruitment models were fitted with fixed values of h, and $S_0 = SPR(0) * R_0$, where R_0 was estimated given posterior stock-recruit data (i.e. SSB and recruit abundance) from the assessment outputs for 2022. Stock-recruit data was summarized as medians of posterior outputs, the median data were fit to the stock-recruit functions, and a single value for R_0 was derived. SPR was defined using the median of posterior LHC values within the SPR function.

Stock-recruit models were fit assuming constant values of steepness, where h = 0.9, 0.75, 0.7, 0.65, & 0.5 for both the BH and RK models, giving 10 different model fits. Root-mean-square-error (RMSE) and AIC for each model were calculated to determine goodness of fit. Maximum Sustainable Yield (MSY) RPs were derived using a standard equilibrium yield analysis (see Appendix B). RPs required for the precautionary approach (PA) leaf HCR), i.e. B_{lim}, $B_{trigger}$, and F_{target}, were also derived for each model to supplement upcoming projection work. Additionally, RPs required for the leaf HCR were used to plot the PA status for each respective fitted SRR model and also to compare relative benchmarks against the previous estimates of $B_{lim} = 14564$ and $SSB_{2022} = 29545$. RPs were derived using an age-20 truncation in per-recruit metrics instead of a plus group. Values derived from the per-recruit analyses using the median of full timeseries LHCs and a 3-year average of LHCs. Fits for stock-recruit curves and RP estimates are provided in a similar format as in Gullage et al. (2024).

Results

RMSEs using the 3-year average LHCs are lower than fits with the full timeseries LHCS in most cases, with some exceptions, and AICs are comparable, although fits using the 3-year average LHCs are slightly lower. The Ricker SRR with steepness h = 0.9 was the fit with the lowest AIC for the 3-year average LHCs, and the optimal (h-estimated) RK fit has the lowest AIC for the full timeseries LHCs and the lowest RMSE for both sets of LHCs. In general, RPs estimated using the 3-year average LHCs are lower than those derived with the full timeseries LHCs.

Tables

Table 1. RMSE and AIC for each model fit (bolded values indicate the model of best fit) derived from 3-year average LHCs.

SRR	h	log(R_0)	Std (logR_0)	RMSE	AIC
BH	0.90	11.73	0.18	93,910	879
BH	0.75	11.89	0.20	90,663	877
BH	0.70	11.92	0.21	90,832	877
BH	0.65	11.96	0.23	91,395	877
BH	0.50	12.06	0.34	95,057	880
RK	**0.90**	**11.60**	**0.14**	**86,015**	**873**
RK	0.75	11.65	0.17	88,212	875

SRR	h	log(R_0)	Std (logR_0)	RMSE	AIC
RK	0.70	11.67	0.18	89,262	876
RK	0.65	11.69	0.20	90,482	877
RK	0.50	11.79	0.30	95,222	880

Table 2. RMSE and AIC for each model fit (bolded values indicate the model of best fit) derived from full timeseries LHCs.

SRR	h	log(R_0)	Std (logR_0)	RMSE	AIC
BH	0.90	11.81	0.18	92,084	878
BH	0.75	12.00	0.23	91,225	877
BH	0.70	12.05	0.25	92,221	878
BH	0.65	12.11	0.29	93,579	879
BH	0.50	12.38	0.55	99,214	883
RK	**0.90**	**11.53**	**0.20**	**90,740**	**877**
RK	0.75	11.63	0.27	94,015	879
RK	0.70	11.68	0.30	95,298	880
RK	0.65	11.74	0.35	96,681	881
RK	0.50	12.06	0.67	101,479	884

Table 3. RMSE and AIC for each model fit based on the optimal h derived from 3-year average LHCs.

SRR	h	log(R_0)	Std (h)	Std (logR_0)	RMSE	AIC
BH	0.744	11.89	0.161	0.25	90,660	879
RK	1.091	11.55	0.257	0.14	85,078	874

Table 4. RMSE and AIC for each model fit based on the optimal h derived from full timeseries LHCs.

SRR	h	$log(R_0)$	Std (h)	Std ($logR_0$)	RMSE	AIC
BH	0.817	11.92	0.127	0.27	90,660	879
RK	1.534	11.30	0.361	0.16	85,078	874

Table 5. MSY Reference Points for each model fit derived from 3-year average LHCs.

SRR	h	F_{MSY}	B_{MSY}	MSY	B_{lim}	$B_{trigger}$	F_{target}
BH	0.90	0.190	23,486	14,421	7,046	17,614	0.162
BH	0.75	0.126	34,357	14,376	10,307	25,768	0.107
BH	0.70	0.112	37,627	14,035	11,288	28,220	0.095
BH	0.65	0.099	40,925	13,588	12,278	30,694	0.084
BH	0.50	0.065	52,141	11,633	15,642	39,106	0.055
RK	0.90	0.136	36,705	16,432	11,012	27,529	0.115
RK	0.75	0.109	39,744	14,514	11,923	29,808	0.093
RK	0.70	0.100	40,955	13,805	12,286	30,716	0.085
RK	0.65	0.092	42,317	13,056	12,695	31,738	0.078
RK	0.50	0.064	48,101	10,536	14,430	36,076	0.054

Table 6. MSY Reference Points for each model fit derived from full timeseries LHCs.

SRR	h	F_{MSY}	B_{MSY}	MSY	B_{lim}	$B_{trigger}$	F_{target}
BH	0.90	0.267	45,273	16,558	13,582	33,955	0.227
BH	0.75	0.198	63,809	17,484	19,143	47,857	0.168
BH	0.70	0.179	70,287	17,504	21,086	52,715	0.152
BH	0.65	0.162	77,525	17,475	23,257	58,144	0.137
BH	0.50	0.112	113,294	17,889	33,988	84,970	0.095
RK	0.90	0.220	54,582	16,532	16,375	40,936	0.187
RK	0.75	0.181	61,939	15,598	18,582	46,454	0.154
RK	0.70	0.168	65,469	15,321	19,641	49,101	0.143
RK	0.65	0.155	69,973	15,095	20,992	52,480	0.131
RK	0.50	0.111	98,901	15,505	29,670	74,176	0.095

Table 7. MSY Reference Points for each model fit based on the optimal h derived from 3-year average LHCs.

SRR	h	F_{MSY}	B_{MSY}	MSY	B_{lim}	$B_{trigger}$	F_{target}
BH	0.744	0.125	34,730	14,343	10,419	26,048	0.106
RK	1.091	0.169	33,590	18,502	10,077	25,192	0.144

Table 8. MSY Reference Points for each model fit based on the optimal h derived from full timeseries LHCs.

SRR	h	F_{MSY}	B_{MSY}	MSY	B_{lim}	$B_{trigger}$	F_{target}
BH	0.817	0.225	55,697	17,299	16,709	41,772	0.192
RK	1.534	0.300	47,824	19,577	14,347	35,868	0.255

Figures

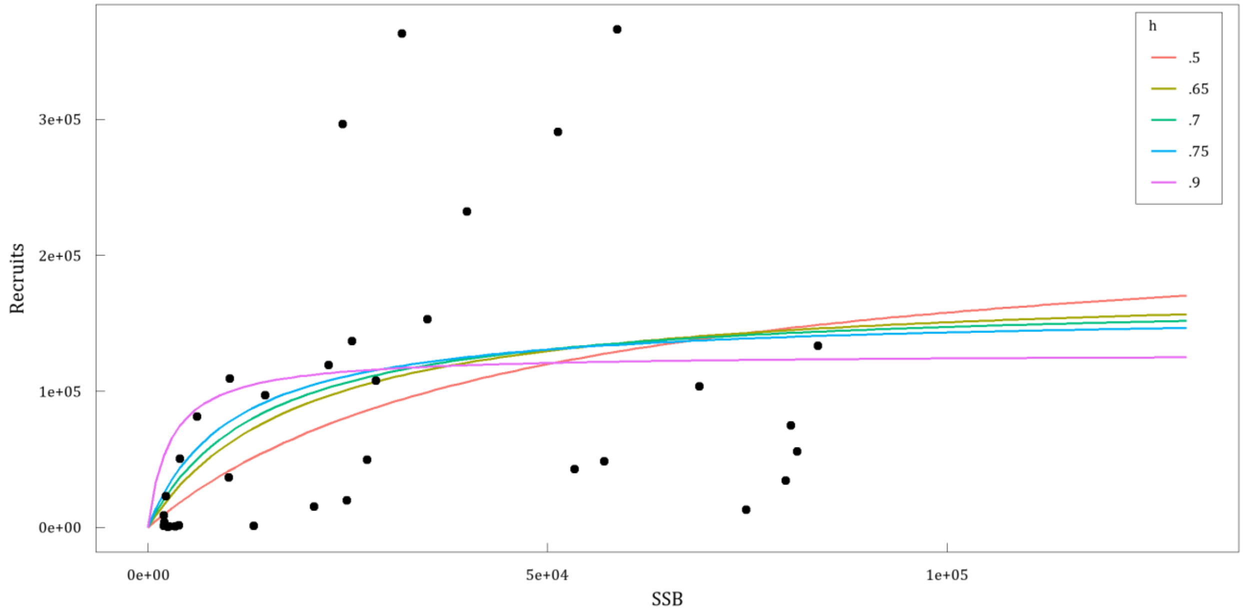

Figure 1. Model fits for all fixed steepness values for the Beverton-Holt SRR derived from 3-year average LHCs.

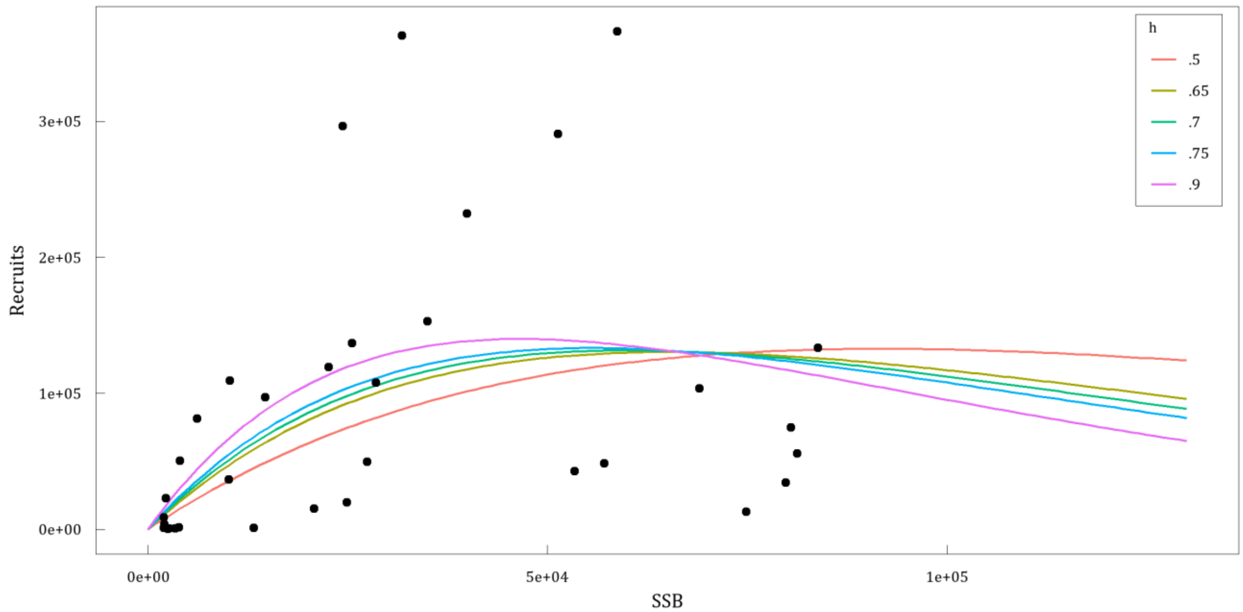

Figure 2. Model fits for all fixed steepness values for the Ricker SRR derived from 3-year average LHCs.

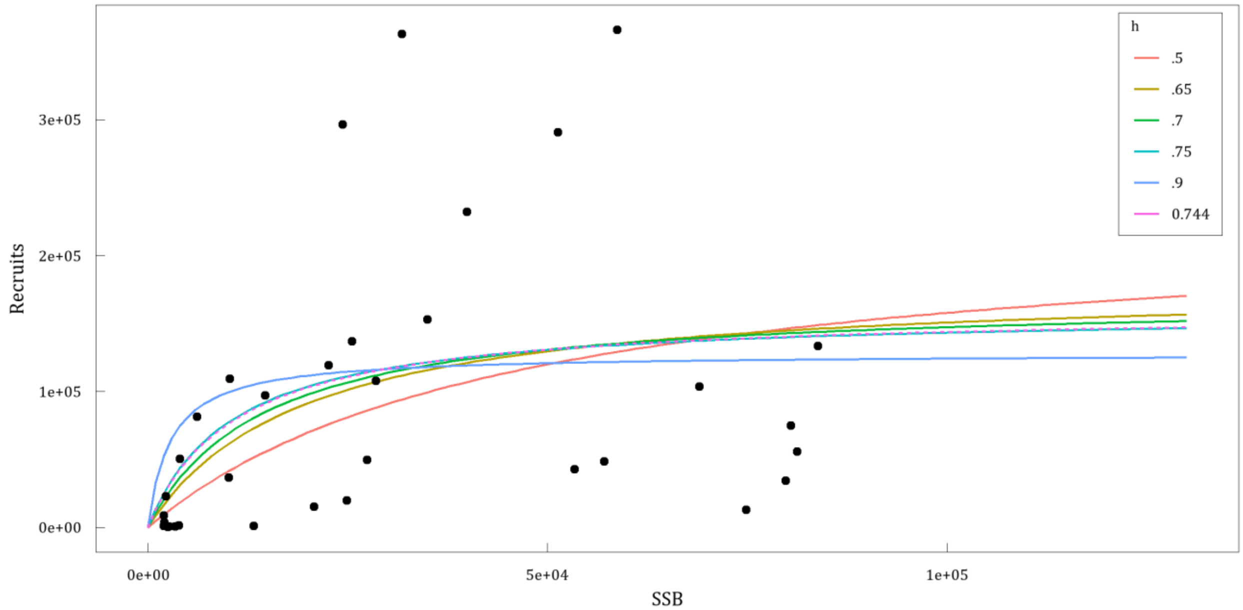

Figure 3. Model fits for all fixed steepness values (solid) compared to the optimal steepness (dashed) for the Beverton-Holt SRR derived from 3-year average LHCs.

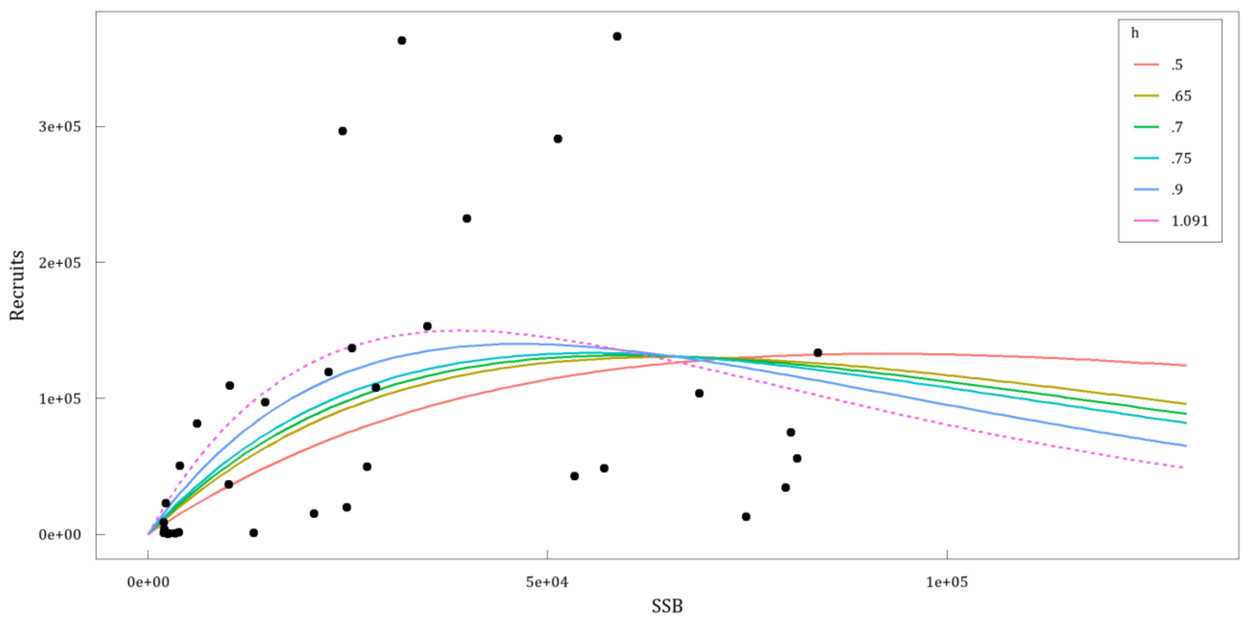

Figure 4. Model fits for all fixed steepness values (solid) compared to the optimal steepness (dashed) for the Ricker SRR derived from 3-year average LHCs.

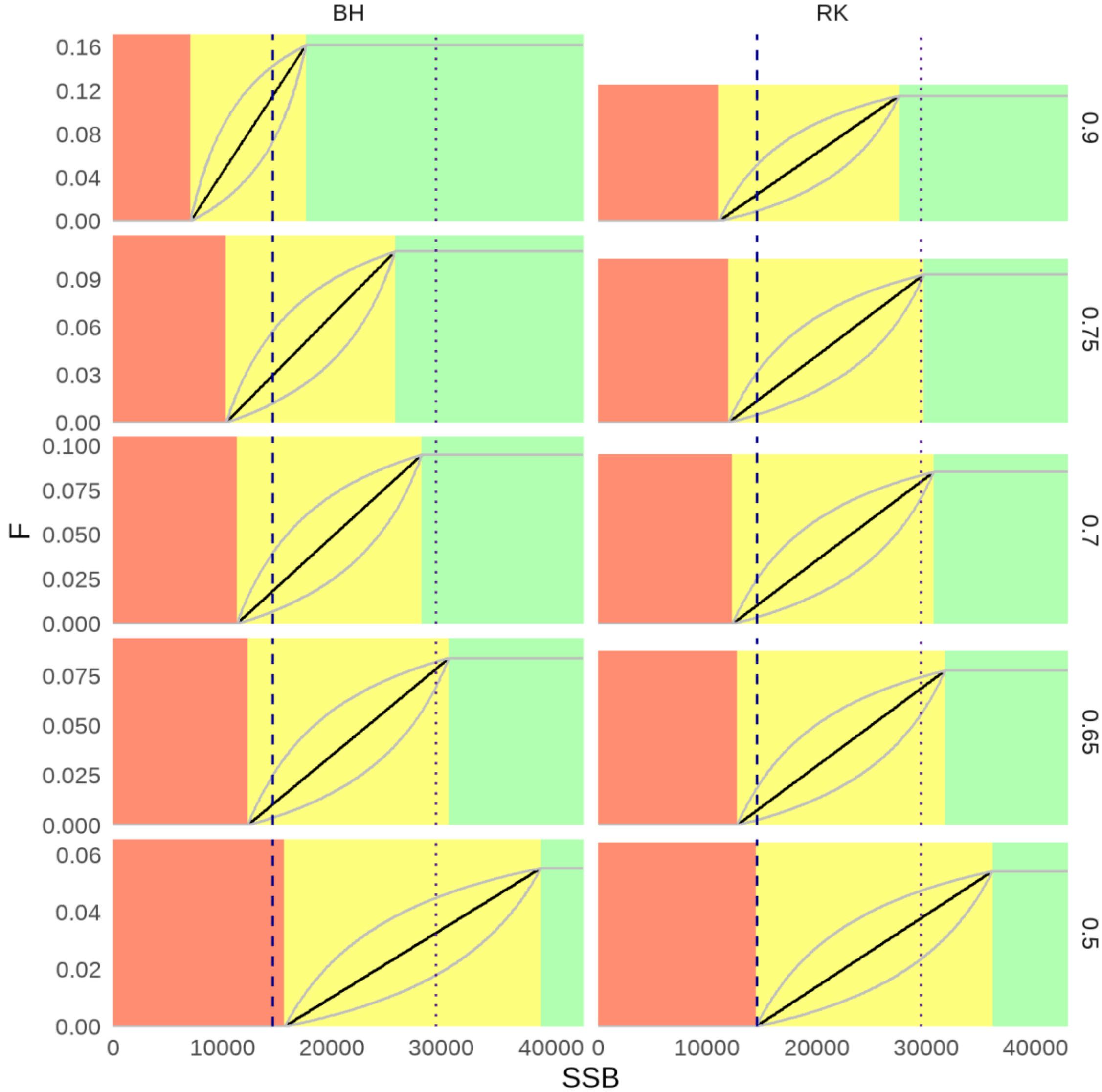

Figure 5. Precautionary Approach harvest control rule leaves based on reference points (i.e. B_{lim}, $B_{trigger}$, and F_{target}) for each stock-recruit curve fit (SRR by column, steepness by row) derived from 3-year average LHCs. The dashed (blue) vertical line is the original B_{lim} = 14564, and the dotted (purple) vertical line is SSB_{2022} = 29545.

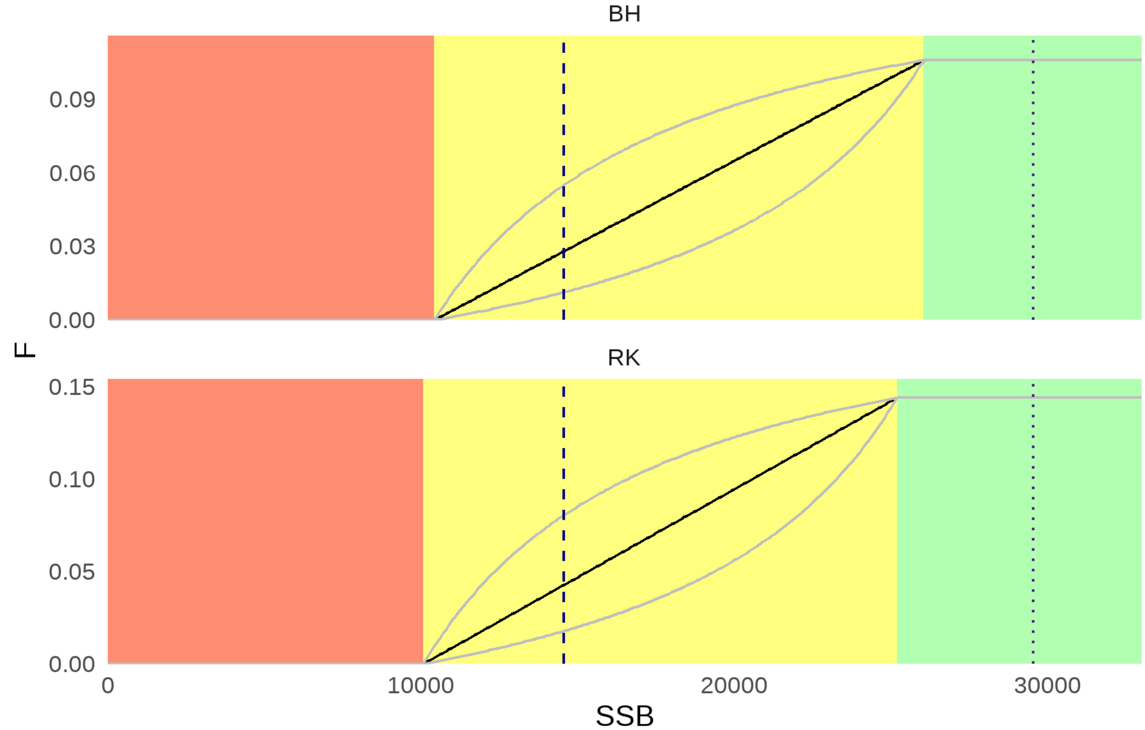

Figure 6. Precautionary Approach harvest control rule leaves based on reference points (i.e. B_{lim}, $B_{trigger}$, and F_{target}) for both of the optimal stock-recruit curve fits derived from 3-year average LHCs. The dashed (blue) vertical line is the original B_{lim} = 14564, and the dotted (purple) vertical line is SSB_{2022} = 29545.

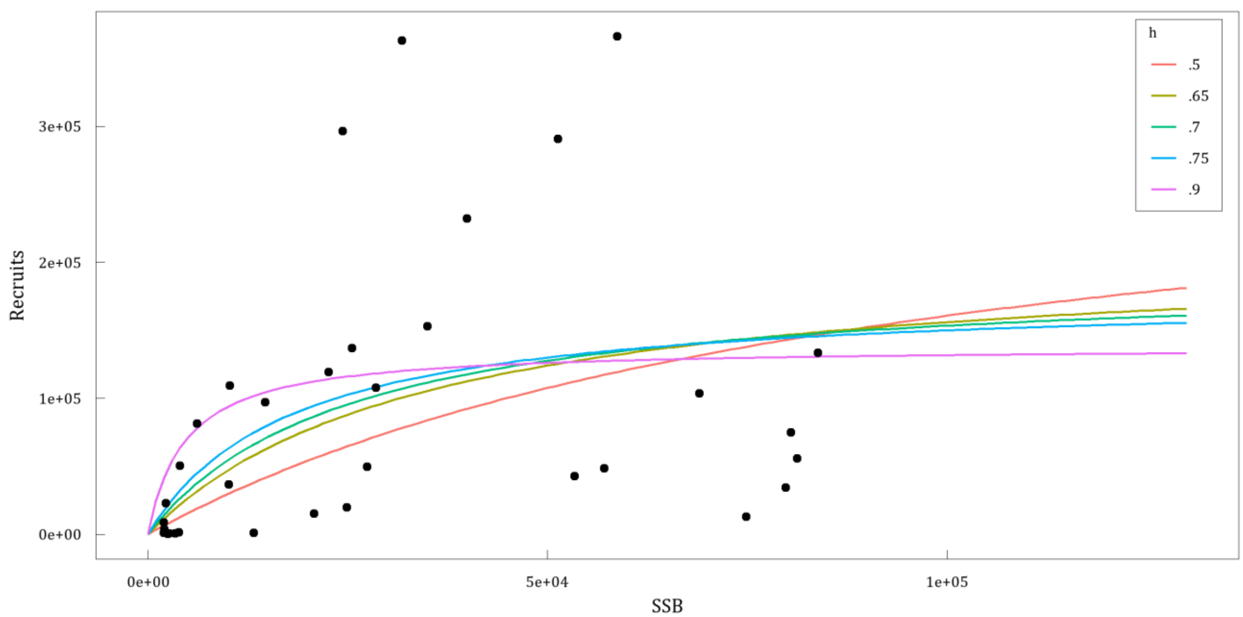

Figure 7. Model fits for all fixed steepness values for the Beverton-Holt SRR derived from full timeseries LHCs.

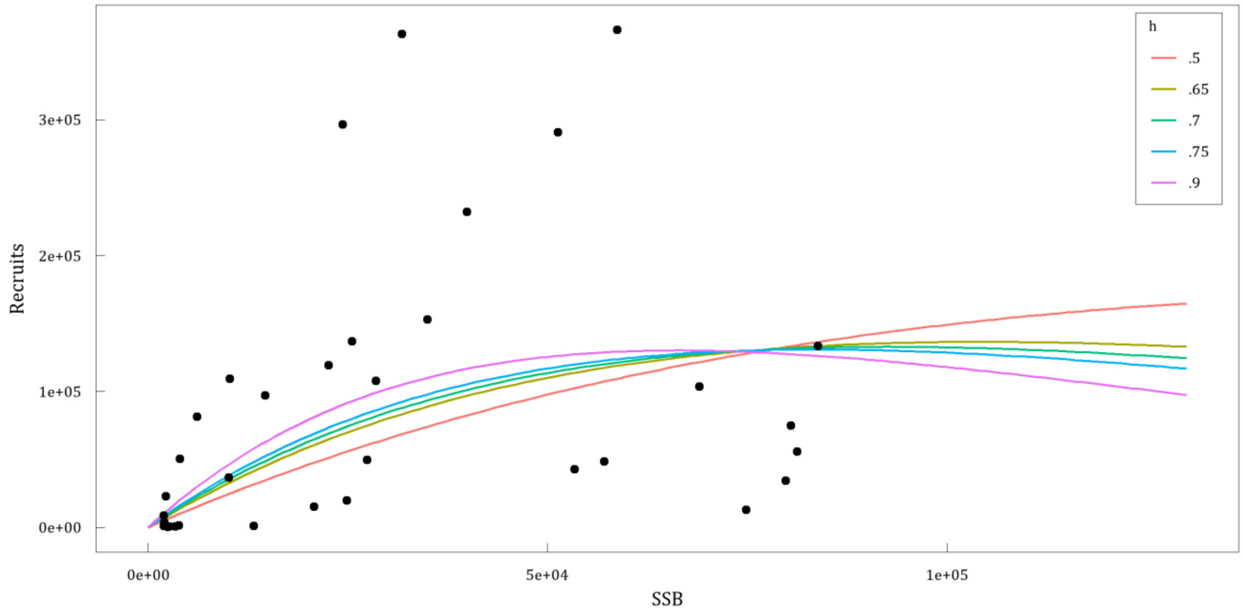

Figure 8. Model fits for all fixed steepness values for the Ricker SRR derived from full timeseries LHCs.

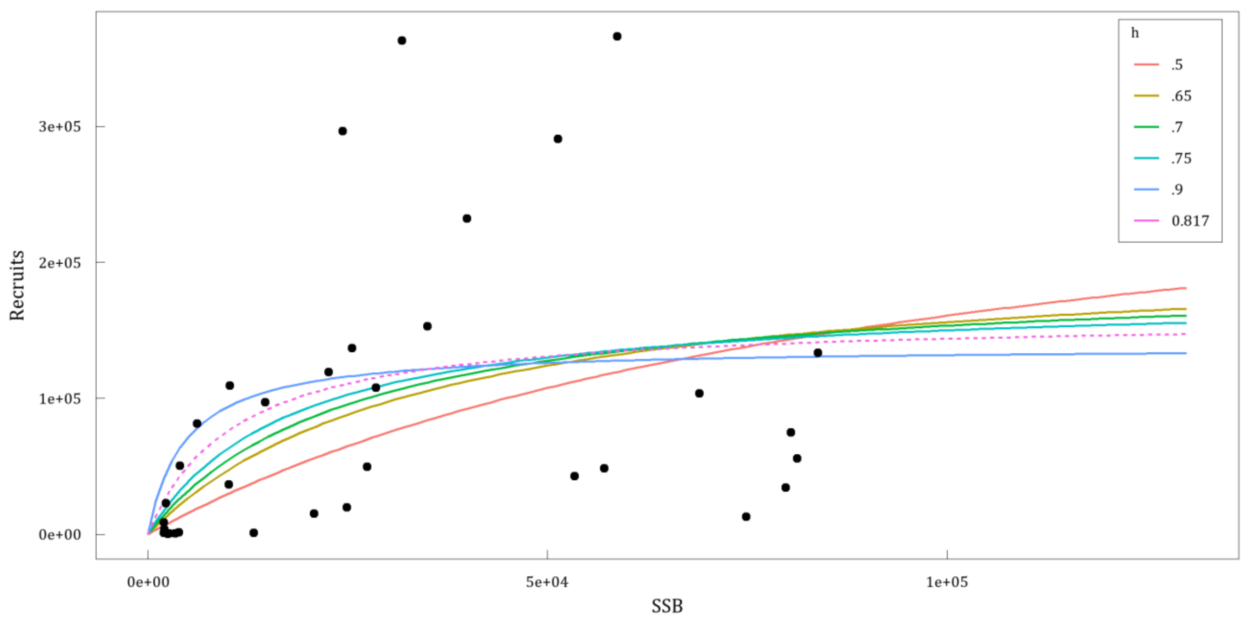

Figure 9. Model fits for all fixed steepness values (solid) compared to the optimal steepness (dashed) for the Beverton-Holt SRR derived from full timeseries LHCs.

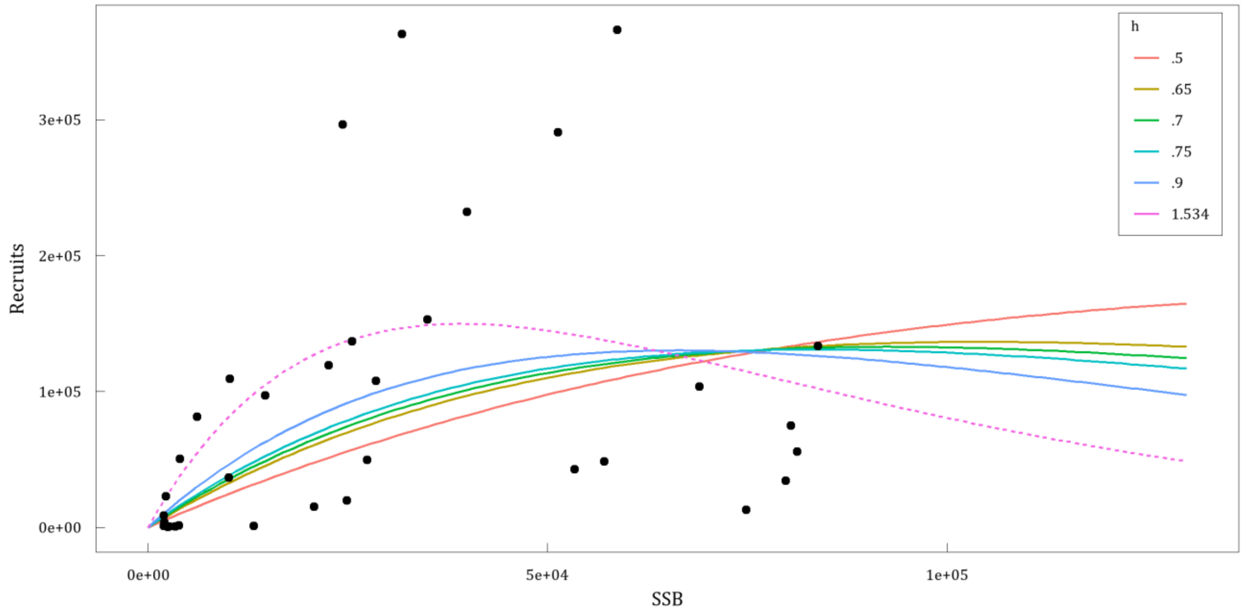

Figure 10. Model fits for all fixed steepness values (solid) compared to the optimal steepness (dashed) for the Ricker SRR derived from full timeseries LHCs.

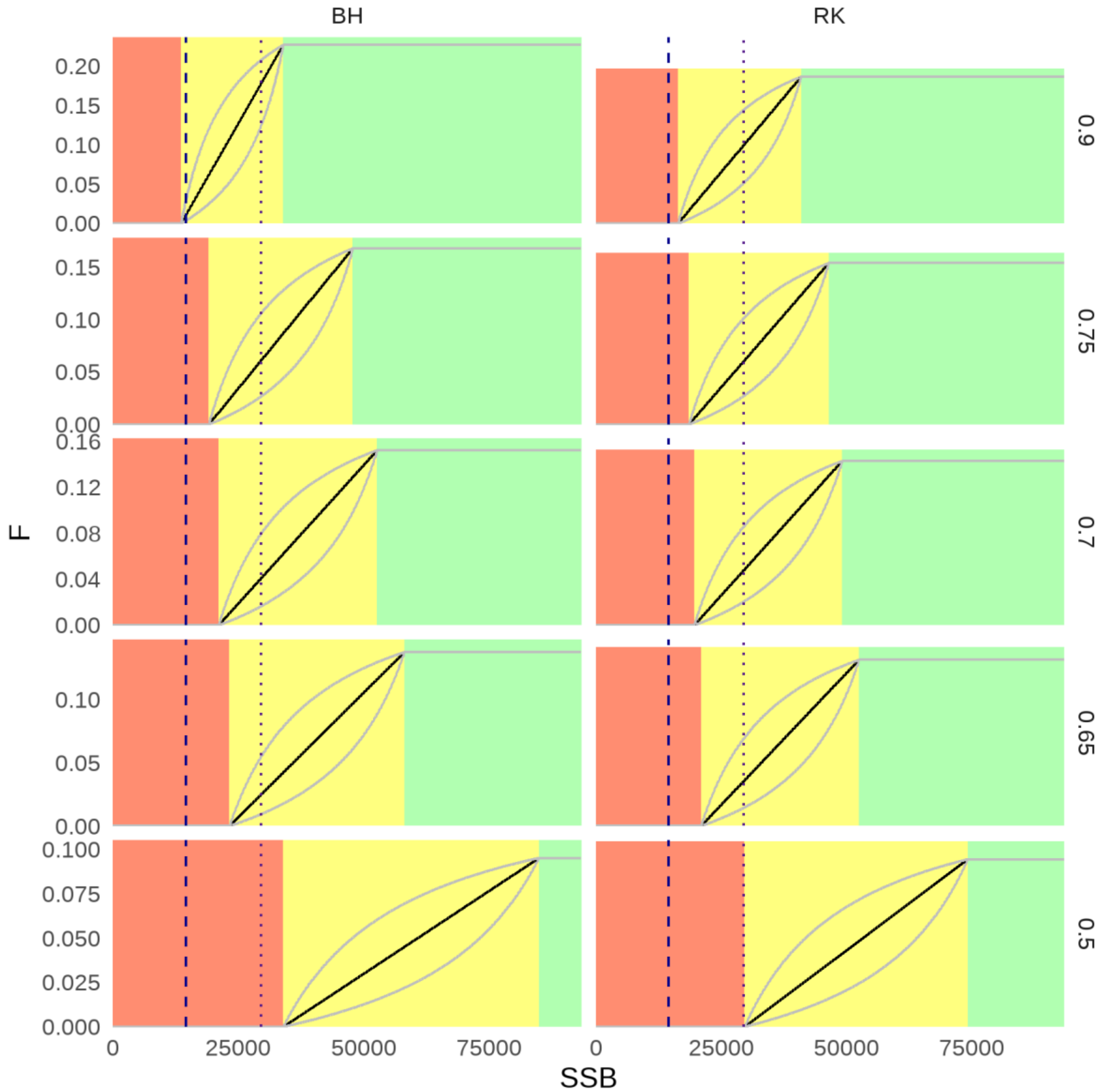

Figure 11. Precautionary Approach harvest control rule leaves based on reference points (i.e. B_{lim}, $B_{trigger}$, and F_{target}) for each stock-recruit curve fit (SRR by column, steepness by row) derived from full timeseries LHCs. The dashed (blue) vertical line is the original B_{lim} = 14564, and the dotted (purple) vertical line is SSB_{2022} = 29545.

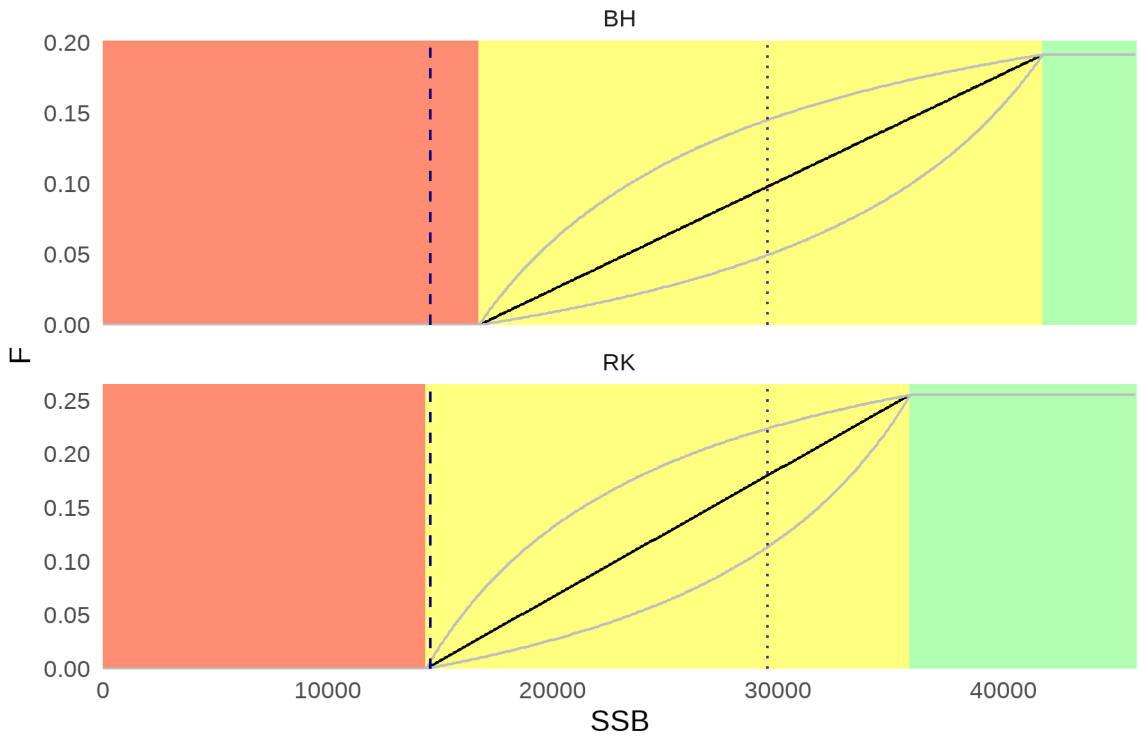

Figure 12. Precautionary Approach harvest control rule leaves based on reference points (i.e. B_{lim}, $B_{trigger}$, and F_{target}) for both of the optimal stock-recruit curve fits derived from full timeseries LHCs. The dashed (blue) vertical line is the original B_{lim} = 14564, and the dotted (purple) vertical line is SSB_{2022} = 29545.

References

Gullage, N., Kumar, R., and Varkey, D. A. (2024). Analysis of stock-recruitment relationships for NAFO div. 3M cod in preparation for MSE simulations. *NAFO SCR Doc*, *24/XXX*.

Kumar, R., Varkey, D. A., and Gullage, N. (2024). Management strategy evaluation (MSE) of cod in NAFO div. 3M to test the proposed NAFO precautionary approach framework (NAFO-PAF). *NAFO SCR Doc*, *24/043*.

Miller, T. J., and Brooks, E. N. (2021). Steepness is a slippery slope. *Fish and Fisheries*, *22*(3), 634–645.

Appendix A

The spawner-per-recruit is defined as a function which evaluates the ratio of spawning biomass to recruits for a given level of fishing mortality, where

$$SPR(f) = \sum_a M_a \, W_a^b e^{-\sum_j^{a-1} z_j},$$

where M_a is maturity-at-age, W_a^b is the beginning-of-year (i.e. stock) weight-at-age, and the exponentiated term is the cumulative sum of total mortalities up to a given age, as indicated by a standard cohort equation, which gives a ratio of abundance-at-age to recruit abundance. here, Z_j is a function of natural, M_a, and fishing, F_a, mortality where fishing mortality is assumed to be a function of selectivity, $F_a = f \cdot sel_a$, and f is constant.

We derive SPR (and YPR) using the median values across posterior samples and years for maturities-at-age and natural mortalities-at-age for the full timeseries LHCs. For the 3-year average LHCs, we use the median across samples but average across the 3 most recent years in the timeseries (i.e., 2020-2022). Values for selectivity and weights-at-age are constant across posterior samples, and are applied as median across respective years, only.

Appendix B

Maximum sustainable yield (MSY) reference points (RPs) are defined as the global optimum of an equilibrium yield curve with respect to *f*. Equilibrium Yield is defined as the long-term stable state of yield from a population that is fished at a constant harvest rate, defined as

$$Y_{eq}(f) = YPR(f)R_{eq},$$

where R_{eq} is the recruitment produced by a stock-recruit relationship for every year at equilibrium, and YPR is the yield-per-recruit.

The yield-per-recruit is defined as a function which evaluates the ratio of yield to recruits for a given level of fishing mortality, where

$$YPR(f) = \sum_a P_a \, W_a^m e^{-\sum_j^{a-1} z_j},$$

where W_a^m is the mid-year (i.e. catch) wight-at-age, and

$$P_a = \frac{F_a}{Z_a}(1 - exp(-Z_a)),$$

is the proportion of catches take for age group a (as in the Baranov catch equation).

At equilibrium, we assume that the age structure of a population across years is equivalent to the age structure of a population within a yearfor example, at equilibrium the abundance of age 3 fish for one year is equivalent to the abundance of age 3 fish for the next year and every subsequent year. This is defined as follows,

$$S_{eq} = SPR(f)R_{eq},$$

where S_{eq} is the SSB for every year at equilibrium. Recruitment is assumed to be function of SSB, and so we can say

$$S_{eq} = SPR(f)R(S_{eq}),$$

and equilibrium SSB can derived by solving the above equation for S_{eq} for a given stock-recruit relationship, where S_{eq} is a function of *f*. Equations for equilibrium SSB are not derived explicitly for the stock-recruit functions used herein, and values are instead solved for numerically using `uniroot` in R for each parametrization. Equilibrium recruitment can be defined as a function of *f* through the equation for equilibrium SSB, such that $R_{eq}(f) = R\left(S_{eq}(f)\right)$. Equilibrium yield can then be optimized with respect to *f*. The value of *f* which optimizes Y_{eq} is F_{MSY}. MSY is the equilibrium yield at F_{MSY}, and B_{MSY} is the equilibrium SSB at F_{MSY}. Equilibrium yields are optimized in R using the `optim` function.